ELEC 225 **Circuit Theory I** **Fall 2013**

Review Topics for Exam #2

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. You should be familiar with the topics on the previous review sheet in addition to those listed below.

Although every effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for your exam.

Linearity and superposition

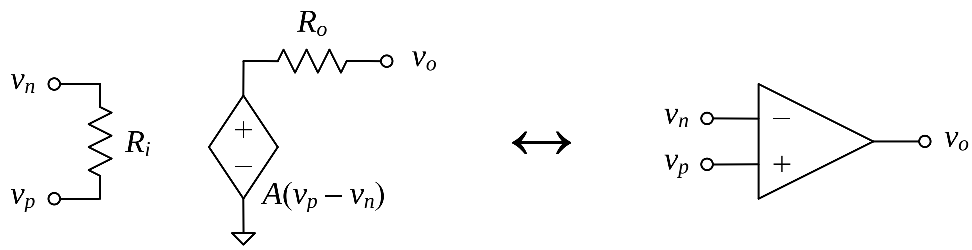
- two conditions of linearity
  - the response (a given circuit voltage or current) to a collection of stimuli (independent voltage or current sources) is equal to the sum of the individual responses to those stimuli
  - if an excitation (an independent voltage or current source) is scaled by a constant K, then the response (the part of a voltage or current due to that source) is also scaled by K
- linearity: The portion of a voltage/current somewhere in a circuit due to a specific indep. voltage/current source is directly proportional to the value of that source.
- principle of superposition: Any voltage or current in a circuit is a weighted sum of the contributions from the individual *independent* sources driving the circuit; i.e., any voltage or current in a circuit can be expressed as a linear combination of independent voltage and current source values. For example, if v_{s1}, v_{s2}, i_{s1}, and i_{s2} are all independent sources, then any voltage v in the circuit can be expressed as

  $$v = K_1 v_{s1} + K_2 v_{s2} + K_3 i_{s1} + K_4 i_{s2},$$

  where K_1 through K_4 are constant coefficients that do not depend on any current or voltage value (but they might be functions of resistor values, gains of dependent sources, numerical constants, etc.).
- Linearity and superposition only apply if all circuit components have *linear* voltage-to-current relationships. These kinds of components include:
  - all independent sources (voltage or current)
  - dependent sources (voltage or current) with constant gain parameters
  - resistors, capacitors, and inductors
- procedure to apply superposition:
  - activate one independent source at a time; deactivate all others (i.e., replace indep. voltage sources with shorts and indep. current sources with opens)
  - leave dependent sources alone
  - find desired circuit voltage(s) and/or current(s) due to the active source
  - repeat the above 3 steps for each individual independent source in the circuit
  - add together the components of the desired circuit voltage(s) and/or current(s) due to the individual sources to find the actual (total) voltage(s) and/or currents(s) that results when all of the sources are active

Operational amplifiers

- op-amp equivalent circuit model: A, R_i, R_o; dependent source $A(v_p - v_n)$



  where v_n, v_p, and v_o are node voltages, and the triangle indicates a connection to the reference node. The circuit on the left is a direct equivalent replacement of the device on the right.
- ideal op-amp characteristics
  - infinite open-loop gain A
  - infinite input resistance R_i between input terminals
  - zero output resistance R_o
  - zero current flow into the inverting and noninverting inputs
- negative feedback
  - must be able to trace a circuit path (not through reference node) from output terminal to inverting input terminal; path could be through a single component or a network of components
  - zero voltage drop across inputs (i.e., $v_p - v_n = 0$), called a "virtual short"
  - zero current into/out of input terminals
  - only applies when op-amp operates linearly (i.e., output voltage not being restricted by power supply voltages, output current limit, or slew rate)
- closed-loop voltage gain vs. open-loop voltage gain
- real op-amps w/neg. feedback: $v_p - v_n$ typically in the range of μV
- output voltage limited by power supply voltages (saturation or clipping); output voltage of most real op-amps is 1-2 V away from power supply limits
- output current of op-amp comes from power supply leads; satisfies KCL; i.e., basic op-amp has 5 branches entering/leaving: 2 inputs, 2 power supply leads, output
- analysis of *ideal* op-amp circuits
  - don't have to use equivalent circuit of op-amp
  - nodal analysis is your friend
  - nodal equation at output of op-amp not usually useful (can't relate output current to node voltages)
  - output current can only be found via KCL after circuit has been analyzed
  - most important goal (typically) is closed-loop gain
  - assumption of ideal behavior is often sufficient for good accuracy
  - usually no effect of load resistance on gain
- standard inverting amplifier circuit
- standard noninverting amplifier circuit
- standard voltage follower (special case of noninverting amplifier)
- standard summing amplifier circuit
- standard difference amplifier circuit

- op-amp output current
  - supplied by power supplies
  - can flow into or out of output terminal
  - usually limited by internal protection circuitry (for 741, limit is ~25-40 mA)
  - can't write nodal equation for output node of op-amp because output node is connected to voltage-controlled voltage source; nodal equation at output node is $v_o = A(v_p - v_n)$; in practice, v_o node is usually ignored in analysis
- gain control resistor values and load resistor values
  - all resistances should be large enough to keep output current below limit
  - resistances should be small enough to minimize noise pick-up and changes due to environmental effects (such as dirt and high humidity)
  - values in the 1 kΩ to 1 MΩ range are typical

Sinusoidal voltages and currents (usually called AC)

- standard cosine form: $v(t) = V_m \cos(\omega t + \phi)$
- V_m = amplitude or magnitude (in units of Vpk, if voltage)
- relationship of Vpk (peak) to Vpp (peak-to-peak) units
- ω = radian frequency (in units of rad/s)
- f = linear or cyclic frequency (in units of Hz; in the past, cycles/s)
- ϕ = phase (in units of degrees or radians, but note that ωt is in radians)
- T = period (in units of s); period is time duration of one full cycle
- $\omega = 2\pi f$, $T = 1/f$

Sinusoidal steady-state (SSS) analysis

- A sinusoidal source (the stimulus) causes all of the other voltages and currents in a circuit (the response) to be sinusoidal at the same frequency, but they will not generally have the same magnitude and phase as the source.
- SSS analysis applies to steady-state (long-term) response of a circuit to an applied sinusoidal voltage/current; the transient (short-term) response is ignored.
- advantages of/reasons to study AC (SSS) include:
  - Electrical power worldwide is generated almost exclusively in AC form (usually at 50 or 60 Hz; 400 Hz in aircraft).
  - All radio/wireless devices use AC to generate/detect electromagnetic waves.
  - Many signals produced by sensors (such as microphones) are AC in nature at one frequency, multiple frequencies, or over a continuum of frequencies.

Inductors

- time-varying magnetic field causes voltage to appear across terminals (this voltage is sometimes called the "back emf," where emf stands for "electromotive force")
- unit is the Henry (H)
- current-voltage relationships
  - passive sign convention – use pos. form if i flows into pos. side of v
  - $v(t) = \pm L \dfrac{di(t)}{dt}$
- voltage *leads* current by 90° (ELI in "ELI the ICE man"); the voltage peaks occur 90° *before* the current peaks in a plot of voltage and current vs. time
- equivalent inductance formulas
  - series: $L_{eq} = L_1 + L_2 + \ldots + L_N$
  - parallel: $L_{eq} = \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \ldots + \dfrac{1}{L_N}}$

Capacitors

- unit is the Farad (F)
- current-voltage relationships
  - passive sign convention – use pos. form if *i* flows into pos. side of *v*
  - $i(t) = \pm C \dfrac{dv(t)}{dt}$
- voltage *lags* current by 90° (ICE in "ELI the ICE man"); the voltage peaks occur 90° *after* the current peaks in a plot of voltage and current vs. time
- equivalent capacitance formulas
  - series: $C_{eq} = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots + \dfrac{1}{C_N}}$
  - parallel: $C_{eq} = C_1 + C_2 + \ldots + C_N$

Phasors

- in electrical & computer engineering (ECE), square root of -1 is *j*, not *i*
- definition, voltage example: $v(t) = \mathrm{Re}\{\mathbf{V}e^{j\omega t}\}$; **V** is the phasor
- by convention in ECE, a phasor represents a cosine (not a sine) function; for example, $\mathbf{V} = V_m \angle \phi_v \leftrightarrow V_m \cos(\omega t + \phi_v)$
- magnitude (amplitude) of cosine function = magnitude (modulus) of phasor
- phase of cosine function in time domain (everything added to ωt) = phase of phasor
- Although *impedances* are complex numbers, they are not phasors, because they do not represent sinusoidal signals.
- representations of phasors:
  - polar form (using the angle symbol); example: $\mathbf{V} = 0.3\angle 30°$ V
  - polar form (complex exponential); example: $\mathbf{V} = 0.3e^{j30°} = 0.3e^{j\pi/6}$ V
  - rectangular form; example: $\mathbf{V} = 0.3(\cos 30° + j\sin 30°) = 0.26 + j0.15$ V
  - phasor diagram (vector in complex plane) can also be used
- conversion from one phasor representation to another
- take care when interpreting result of inverse tangent function on calculator

Sinusoidal steady-state AC circuit analysis using phasors

- also known as *frequency-domain* analysis
- impedance, *Z*:
  - resistor: $Z = R$
  - inductor: $Z = j\omega L$
  - capacitor: $Z = \dfrac{1}{j\omega C}$
- Ohm's Law for impedances:
  - $\mathbf{V} = \mathbf{I}Z$
  - voltage and current are phasors (indicated by boldface or by tilde ~)
  - impedance, although complex, is not a phasor (so not boldface/no tilde)
  - note that the use of boldface and/or tilde is not a widespread standard; must pay attention to context
- equivalent impedance formulas
  - series: $Z_{eq} = Z_1 + Z_2 + \ldots + Z_N$

    - parallel: $Z_{eq} = \dfrac{1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \ldots + \dfrac{1}{Z_N}}$
    - same rules as those used for resistors
- circuit analysis techniques applicable in frequency domain:
    - Ohm's law
    - KVL, KCL
    - voltage-divider formula, current-divider formula
    - superposition and linearity
- voltage magnitudes in a circuit with inductors/capacitors can be higher than source voltage(s)
- current magnitudes in a circuit with inductors/capacitors can be higher than source current(s)

<u>Relevant course material:</u>

HW:	#4-#7
Labs:	#3-#5
Textbook:	Secs. 4-1 through 4-7
	Secs. 7-1 through 7-4 (but not admittance or Y-Δ transformation)
Supplements:	Complex Arithmetic Examples on Homework Assignments web page