Linearity and superposition

- Two conditions of linearity
  o The response (a given circuit voltage or current) to a collection of stimuli (independent voltage or current sources) is equal to the sum of the individual responses to those stimuli
  o If an excitation (an independent voltage or current source) is scaled by a constant $K$, then the response (the part of a voltage or current due to that source) is also scaled by $K$

- Linearity: The portion of a voltage/current somewhere in a circuit due to a specific indep. voltage/current source is directly proportional to the value of that source.

- Principle of superposition: Any voltage or current in a circuit is a weighted sum of the contributions from the individual independent sources driving the circuit; i.e., any voltage or current in a circuit can be expressed as a linear combination of independent voltage and current source values. For example, if $v_{s1}$, $v_{s2}$, $i_{s1}$, and $i_{s2}$ are all independent sources, then any voltage $v$ in the circuit can be expressed as

$$v = K_1 v_{s1} + K_2 v_{s2} + K_3 i_{s1} + K_4 i_{s2},$$

where $K_1$ through $K_4$ are constant coefficients that do not depend on any current or voltage value (but they might be functions of resistor values, gains of dependent sources, numerical constants, etc.).

- Linearity and superposition only apply if all circuit components have linear voltage-to-current relationships. These kinds of components include:
  o All independent sources (voltage or current)
  o Dependent sources (voltage or current) with constant gain parameters
  o Resistors, capacitors, and inductors

- Procedure to apply superposition:
  o Activate one independent source at a time; deactivate all others (i.e., replace indep. voltage sources with shorts and indep. current sources with opens)
  o Leave dependent sources alone
  o Find desired circuit voltage(s) and/or current(s) due to the active source
  o Repeat the above 3 steps for each individual independent source in the circuit
  o Add together the components of the desired circuit voltage(s) and/or current(s) due to the individual sources to find the actual (total) voltage(s) and/or currents(s) that results when all of the sources are active
Operational amplifiers
- op-amp equivalent circuit model: $A, R_i, R_o$; dependent source $A(v_p - v_n)$

$$\begin{align*}
R_o & \quad v_o \\
R_i & \quad + \quad A(v_p - v_n) \\
v_p & \quad - \\
v_n & \quad v_o
\end{align*}$$

where $v_n$, $v_p$, and $v_o$ are node voltages, and the triangle indicates a connection to the reference node. The circuit on the left is a direct equivalent replacement of the device on the right.

- ideal op-amp characteristics
  - infinite open-loop gain $A$
  - infinite input resistance $R_i$ between input terminals
  - zero output resistance $R_o$
  - zero current flow into the inverting and noninverting inputs

- negative feedback
  - must be able to trace a circuit path (not through reference node) from output terminal to inverting input terminal; path could be through a single component or a network of components
  - zero voltage drop across inputs (i.e., $v_p - v_n = 0$), called a “virtual short”
  - zero current into/out of input terminals
  - only applies when op-amp operates linearly (i.e., output voltage not being restricted by power supply voltages, output current limit, or slew rate)

- closed-loop voltage gain vs. open-loop voltage gain
- real op-amps w/neg. feedback: $v_p - v_n$ typically in the range of $\mu$V
- output voltage limited by power supply voltages (saturation or clipping); output voltage of most real op-amps is 1-2 V away from power supply limits
- output current of op-amp comes from power supply leads; satisfies KCL; i.e., basic op-amp has 5 branches entering/leaving: 2 inputs, 2 power supply leads, output

- analysis of ideal op-amp circuits
  - don’t have to use equivalent circuit of op-amp
  - nodal analysis is your friend
  - nodal equation at output of op-amp not usually useful (can’t relate output current to node voltages)
  - output current can only be found via KCL after circuit has been analyzed
  - most important goal (typically) is closed-loop gain
  - assumption of ideal behavior is often sufficient for good accuracy
  - usually no effect of load resistance on gain

- standard inverting amplifier circuit
- standard noninverting amplifier circuit
- standard voltage follower (special case of noninverting amplifier)
- standard summing amplifier circuit
- standard difference amplifier circuit
- op-amp output current
  o supplied by power supplies
  o can flow into or out of output terminal
  o usually limited by internal protection circuitry (for 741, limit is ~25-40 mA)
  o can’t write nodal equation for output node of op-amp because output node is connected to voltage-controlled voltage source; nodal equation at output node is \( v_o = A(v_p - v_n) \); in practice, \( v_o \) node is usually ignored in analysis
- gain control resistor values and load resistor values
  o all resistances should be large enough to keep output current below limit
  o resistances should be small enough to minimize noise pick-up and changes due to environmental effects (such as dirt and high humidity)
  o values in the 1 k\( \Omega \) to 1 M\( \Omega \) range are typical

Sinusoidal voltages and currents (usually called AC)
- standard cosine form: \( v(t) = V_m \cos(\omega t + \phi) \)
- \( V_m = \) amplitude or magnitude (in units of Vpk, if voltage)
- relationship of Vpk (peak) to Vpp (peak-to-peak) units
- \( \omega = \) radian frequency (in units of rad/s)
- \( f = \) linear or cyclic frequency (in units of Hz; in the past, cycles/s)
- \( \phi = \) phase (in units of degrees or radians, but note that \( \omega t \) is in radians)
- \( T = \) period (in units of s); period is time duration of one full cycle
- \( \omega = 2\pi f, T = 1/f \)

Sinusoidal steady-state (SSS) analysis
- A sinusoidal source (the stimulus) causes all of the other voltages and currents in a circuit (the response) to be sinusoidal at the same frequency, but they will not generally have the same magnitude and phase as the source.
- SSS analysis applies to steady-state (long-term) response of a circuit to an applied sinusoidal voltage/current; the transient (short-term) response is ignored.
- advantages of/reasons to study AC (SSS) include:
  o Electrical power worldwide is generated almost exclusively in AC form (usually at 50 or 60 Hz; 400 Hz in aircraft).
  o All radio/wireless devices use AC to generate/detect electromagnetic waves.
  o Many signals produced by sensors (such as microphones) are AC in nature at one frequency, multiple frequencies, or over a continuum of frequencies.

Inductors
- time-varying magnetic field causes voltage to appear across terminals (this voltage is sometimes called the “back emf,” where emf stands for “electromotive force”)
- unit is the Henry (H)
- current-voltage relationships
  o passive sign convention – use pos. form if \( i \) flows into pos. side of \( v \)
  o \( v(t) = \pm L \frac{di(t)}{dt} \)
- voltage leads current by 90° (ELI in “ELI the ICE man”); the voltage peaks occur 90° before the current peaks in a plot of voltage and current vs. time
- equivalent inductance formulas
  o series: \( L_{eq} = L_1 + L_2 + \ldots + L_N \)
  o parallel: \( L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}} \)
Capacitors
- unit is the Farad (F)
- current-voltage relationships
  o passive sign convention – use pos. form if \( i \) flows into pos. side of \( v \)
  \[ i(t) = \pm C \frac{dv(t)}{dt} \]
- voltage lags current by 90° (ICE in “ELI the ICE man”); the voltage peaks occur 90° after the current peaks in a plot of voltage and current vs. time
- equivalent capacitance formulas
  o series: \( C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}} \)
  o parallel: \( C_{eq} = C_1 + C_2 + \ldots + C_N \)

Phasors
- in electrical & computer engineering (ECE), square root of –1 is \( j \), not \( i \)
- definition, voltage example: \( v(t) = \text{Re}\{V e^{j\omega t}\} \); \( V \) is the phasor
- by convention in ECE, a phasor represents a cosine (not a sine) function; for example, \( V = V_m \cos(\omega t + \phi_v) \)
- magnitude (amplitude) of cosine function = magnitude (modulus) of phasor
- phase of cosine function in time domain (everything added to \( \omega t \)) = phase of phasor
- Although impedances are complex numbers, they are not phasors, because they do not represent sinusoidal signals.
- representations of phasors:
  o polar form (using the angle symbol); example: \( V = 0.3 \angle 30^\circ \) V
  o polar form (complex exponential); example: \( V = 0.3 e^{j30^\circ} = 0.3 e^{j\pi/6} \) V
  o rectangular form; example: \( V = 0.3(\cos 30^\circ + j \sin 30^\circ) = 0.26 + j0.15 \) V
  o phasor diagram (vector in complex plane) can also be used
- conversion from one phasor representation to another
- take care when interpreting result of inverse tangent function on calculator

Sinusoidal steady-state AC circuit analysis using phasors
- also known as frequency-domain analysis
- impedance, \( Z \):
  o resistor: \( Z = R \)
  o inductor: \( Z = j\omega L \)
  o capacitor: \( Z = \frac{1}{j\omega C} \)
- Ohm’s Law for impedances:
  o \( V = IZ \)
  o voltage and current are phasors (indicated by boldface or by tilde ~)
  o impedance, although complex, is not a phasor (so not boldface/no tilde)
  o note that the use of boldface and/or tilde is not a widespread standard; must pay attention to context
- equivalent impedance formulas
  o series: \( Z_{eq} = Z_1 + Z_2 + \ldots + Z_N \)
parallel: \( Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_N}} \)

- same rules as those used for resistors
- circuit analysis techniques applicable in frequency domain:
  - Ohm’s law
  - KVL, KCL
  - voltage-divider formula, current-divider formula
  - superposition and linearity
- voltage magnitudes in a circuit with inductors/capacitors can be higher than source voltage(s)
- current magnitudes in a circuit with inductors/capacitors can be higher than source current(s)

Relevant course material:

HW: #4-#7
Labs: #3-#5
Textbook: Secs. 4-1 through 4-7
  Secs. 7-1 through 7-4 (but not admittance or Y-Δ transformation)
Supplements: Complex Arithmetic Examples on Homework Assignments web page