ECEG 351 Electronics II Spring 2024

Homework Assignment #5 – due via Moodle at 11:59 pm on Friday, Mar. 1, 2024

***Instructions, notes, and hints*:**

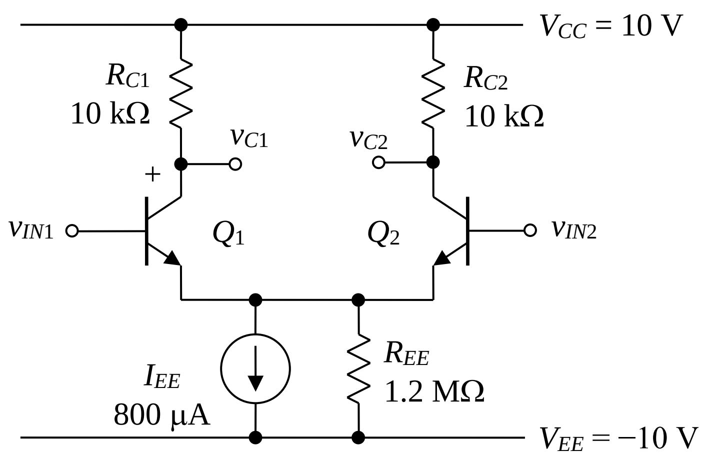
You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

Unless otherwise specified, you may assume that all BJTs are at room temperature, the emission coefficient $\eta = 1$, $V_{BE} = 0.7$ V (quiescent value), and $V_{CE}|_{\text{sat}} = 0.2$ V. If the Early voltage V_A is not specified, you may ignore its effects. For now, unless otherwise specified, capacitors can be assumed to have values large enough that they act as shorts at the operating frequency.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Shown below is a standard BJT differential amplifier with a mirror current I_{EE} and mirror output resistance R_{EE}. Suppose that the collector resistors are significantly mismatched (i.e., they differ in value by ΔR_C; the values for R_{C1} and R_{C2} shown below are the nominal values) but that the mismatch between the transistors is negligible. If the output is taken differentially, the resulting CMRR is 40 dB higher than when a single-ended output is used. Find the collector resistance mismatch ratio $\Delta R_C/R_C$ expressed as a percentage that accounts for the 40 dB difference in CMRRs between the single-ended and differential output cases. For the BJTs, $V_{BE}|_{\text{on}} = 0.7$ V, $V_{CE}|_{\text{sat}} = 0.3$ V, and $\beta = 150$. The Early effect can be ignored (i.e., $r_o \to \infty$.) You may assume that the emission coefficient $n = 1$ and the thermal voltage V_T = 25 mV. You may also assume that $\alpha \approx 1$, where $\alpha = \beta/(\beta + 1)$.



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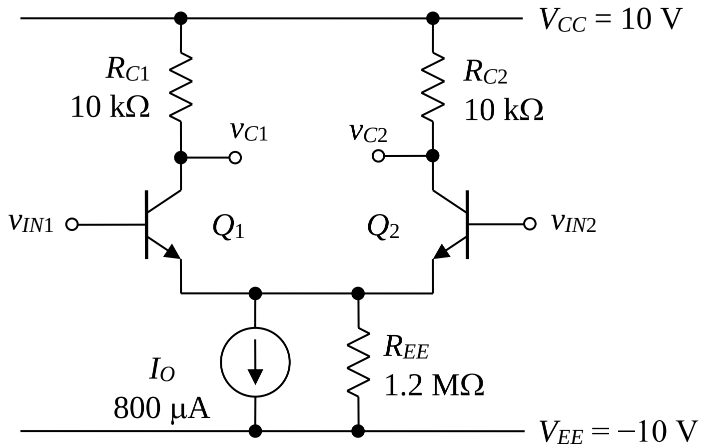
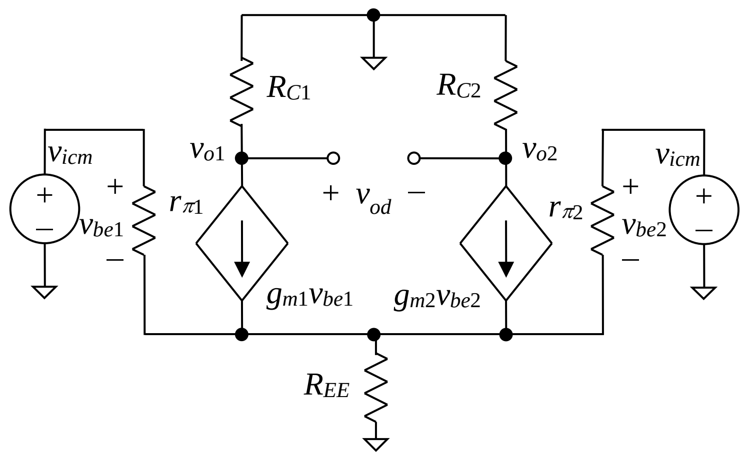
2. Shown below is the diff amp considered in the previous problem. The current mirror is a Widlar type with a quiescent voltage of approximately 0.11 V across the emitter resistor (commonly labeled R_E, not R_{EE} shown in the diagram below) that is in series with the pass transistor. Find the range of *quiescent* base voltages V_{IN1} and V_{IN2} for which the circuit operates as intended. That is, find the common-mode range of the circuit. Note that proper operation requires that Q_1 and Q_2 remain in the active region and that the voltage across the current mirror be above the compliance limit. You may assume that the BJTs have the same parameter values as in the previous problem.



3. Consider the small-signal model shown below of a BJT differential amplifier being fed by a small-signal common-mode voltage. The collector resistors are mismatched by ΔR_C and the transconductances are mismatched by Δg_m. Assume that the actual values can be expressed as shown below right, where R_C and g_m are the average values, respectively, of the collector resistances and the transconductances. Prove that the single-ended common-mode gains relative to the two collector terminals are given by the expressions shown farther below right.



$$R_{C1} = R_C + \frac{\Delta R_C}{2}, \quad R_{C2} = R_C - \frac{\Delta R_C}{2}$$

$$g_{m1} = g_m + \frac{\Delta g_m}{2}, \quad g_{m2} = g_m - \frac{\Delta g_m}{2}$$

$$\frac{v_{o1}}{v_{icm}} = -\frac{g_{m1} R_{C1}}{2 g_m R_{EE}}$$

$$\frac{v_{o2}}{v_{icm}} = -\frac{g_{m2} R_{C2}}{2 g_m R_{EE}}$$

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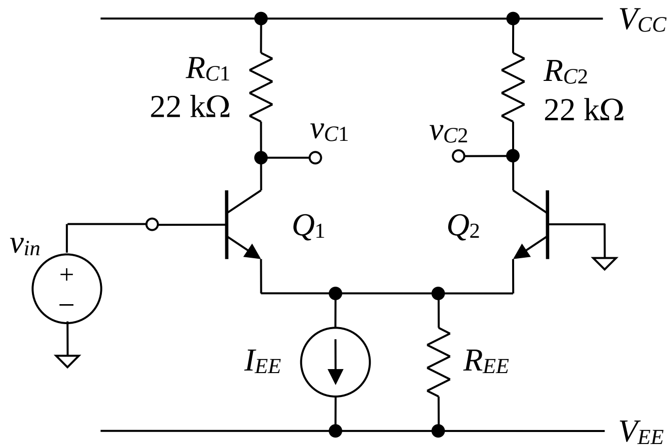
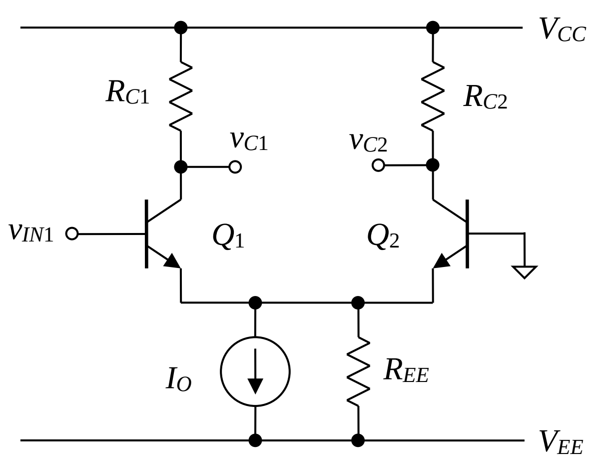
4. The BJT-based differential amplifier shown below will use the indicated collector resistor values to achieve a differential gain A_d of around 180 V/V. The quiescent input voltage is zero, and the current source driving the diff amp has an output resistance R_{EE} that is so large that it can be ignored for most analysis tasks. You may also ignore the Early effect for the first three parts below. The BJT parameter values given in the “Instructions, notes, and hints” at the beginning of this assignment apply.

a. Find the required value of g_m for the two transistors and the associated mirror current I_{EE} to achieve the specified gain.
b. Find the quiescent voltages across R_{C1} and R_{C2}.
c. Suppose that v_{in} is a sinusoidal voltage limited to a peak value of 5.0 mV. Find the lowest value of V_{CC} that allows Q_1 and Q_2 to operate in the active region at all times.
d. Suppose that Q_1 and Q_2 each have an Early voltage of V_A = 100 V. Find the differential gain with the Early effect included.



5. Depicted below is a standard diff amp with single-ended input. Suppose that the quiescent input voltage is to be 0 V and that the quiescent collector voltages are also to be 0 V to ensure that Q_1 and Q_2 remain in the active region. Find the maximum possible voltage gain magnitude for single-ended outputs $A_{d,se1}$ (or $A_{d,se2}$) under these conditions if the power supply voltages are a) ±15 V and b) ±5 V. Assume that all BJTs are matched with emission coefficient $\eta = 1$, $V_{BE} = 0.7$ V (quiescent), $V_T = 25$ mV, and $V_{CE}|_{\text{sat}} = 0.2$ V. Also assume that the Early voltage $V_A \rightarrow \infty$.



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***Ungraded Problems*:**

The following problems will not be graded. However, you should attempt to solve them on your own and then check the solutions. Try not to give up too quickly if you struggle to solve any of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. A major advantage of the Widlar current source is that it has a much higher output resistance than the simple mirror. However, the increase is relatively modest. The derivation shown below left shows how the expression for the Norton equivalent resistance can be modified to gain a better understanding of the limitations. Because the voltage V_{RE} across R_E should be no more than about 0.1 V for practical reasons, the approximation in the last line of the derivation is valid; that is, $V_{RE}/(nV_T) \approx 4$ for $V_{RE} = 0.1$ V. One implication of this result is that the reference current I_{REF} in the Widlar mirror must be much larger than the load current I_L to achieve a significant improvement in the output resistance. Suppose that the load current in the circuit shown below is to be about 150 μA and that the mirror's output resistance is to be about 5 times r_{o2}, where r_{o2} is the Early resistance of Q_2. Find the required value of the reference current I_{REF} to meet the specifications. The BJTs are matched and have $100 < \beta < 300$ and $V_A = 100$ V.

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\begin{aligned}
R_{EE} &\approx \left[1 + g_{m2}\left(R_E \| r_{\pi 2}\right)\right] r_{o2} \\
&\approx \left[1 + \frac{I_L}{nV_T}\left(\frac{V_{RE}}{I_L} \middle\| \frac{\beta n V_T}{I_L}\right)\right] r_{o2} \\
&= \left[1 + \left(\frac{V_{RE}}{nV_T} \middle\| \beta\right)\right] r_{o2} \\
&\approx \left(1 + \frac{V_{RE}}{nV_T}\right) r_{o2}
\end{aligned}
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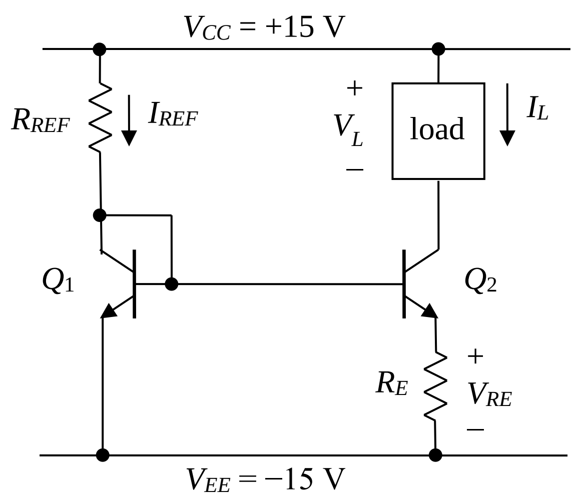

2. In an actual BJT-based differential amplifier, the collector resistors are mismatched by ΔR_C *and* the transconductances are mismatched by Δg_m. Prove that if the output voltage is taken differentially, then the formula shown below gives a good approximation of the common-mode voltage gain magnitude $|A_{cm}|$. The quantities g_m and R_C are the average transconductance and collector resistance, respectively, and R_{EE} is the output resistance of the current mirror.

$$|A_{cm}| \approx \frac{R_C}{2R_{EE}}\left(\frac{\Delta g_m}{g_m} + \frac{\Delta R_C}{R_C}\right)$$

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3. A standard BJT diff amp has been fabricated in an integrated circuit package. The base-emitter junction area of Q_1 is 2.0% larger than that of Q_2, and the values of R_{C1} and R_{C2} are 10.02 kΩ and 9.95 kΩ, respectively. The current mirror has a nominal value of I_{EE} = 2.0 mA, and its output resistance is R_{EE} = 50 kΩ. The differential signal v_{id} expressed below is applied to the input of the amplifier, but also present at both inputs is the common-mode signal v_{icm} expressed below. Find a time-domain expression for the differential-mode output voltage v_{od}, assuming that the differential load resistance is infinite. For the BJTs, $V_{BE}|_{\text{on}}$ = 0.7 V, $V_{CE}|_{\text{sat}}$ = 0.3 V, and β = 150. The Early effect can be ignored (i.e., $r_o \to \infty$.) You may assume that the emission coefficient η = 1 and the thermal voltage V_T = 25 mV.

$$v_{id}(t) = 1.5\cos(240\pi t + 82°)\text{ mV} \qquad v_{icm}(t) = 450\cos(44{,}000\pi t)\text{ mV}$$