Review Topics for Exam #1

Please review the “Exam Policies” section of the Exams page at the course web site. You should especially note the following:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-99.
2. You will be allowed to use one 8.5 × 11-inch two-sided handwritten help sheet. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you later.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible.

Although every effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for your exam.

Electromagnetic spectrum
- allocation of frequencies is determined by international agreement
- choice of frequency for radio service can be dictated by:
  - antenna size (which in turn depends on wavelength)
  - absorption properties of atmosphere or ionospheric refraction
  - other propagation conditions
  - capabilities of current technology
  - government regulations and/or treaty obligations

Permittivity of free space \((\varepsilon_0) = 8.854 \times 10^{-12} \text{ F/m}\)
Permeability of free space \((\mu_0) = 4\pi \times 10^{-7} \text{ H/m}\)

Phasors
- complex numbers that represent sinusoidal functions in time
- one standard definition (the one we use): \(A(t) = \text{Re}\{\tilde{A} e^{j\omega t}\}\)
- in the textbook, phasors are represented by the tilde symbol (~) over the variable
- all phasors are generally complex, but not all complex numbers are phasors (e.g., impedance is not a phasor quantity)
- if \(A = |A| e^{j\phi}\), then \(A(t) = |A| \cos(\omega t + \phi)\); \(A\) and \(\phi\) can be functions of location
- phasors can be functions of any variable (usually spatial variables such as \(x, y\), and/or \(z\)) except time
- \(\frac{\partial A(t)}{\partial t} \leftrightarrow j\omega \tilde{A}\) and \(\frac{\partial^2 A(t)}{\partial t^2} \leftrightarrow (j\omega)^2 \tilde{A} = -\omega^2 \tilde{A}\)
addition and subtraction in time domain are equivalent to addition and subtraction in phasor domain.

- multiplication and division in time domain are not equivalent to multiplication and division in phasor domain.

**Complex arithmetic**

- addition and subtraction, multiplication and division
- complex conjugate (*)
- polar form vs. rectangular form, and conversion between forms
- different types of polar form: $2 \cos 30^\circ = 2e^{j\pi/6}$
- identification of magnitude and phase
- identification of real part and imaginary part
- graphical representation of complex nos. on the complex plane (Im part vs. Re part)
- Euler’s identity: $e^{j\theta} = \cos \theta + j \sin \theta$

- $|z| = \sqrt{zz^*}$ (z is a complex number; superscript * represents conjugation)

- $|e^{j\theta}| = 1$, regardless of the value of $\theta$, where $\theta$ is a real-valued constant or function

- $j = e^{j(0.5\pi \pm 2m\pi)}$, $-j = e^{j(-0.5\pi \pm 2m\pi)}$, and $-1 = e^{j(n\pi \pm 2m\pi)}$, where $n = 0, 1, 2, ...$

**Transmission line fundamentals**

- What constitutes a TEM line? (two long, parallel conductors separated by dielectric; there must be two conductors across which a voltage can be measured; current must be able to flow outward along one conductor and back via the other conductor)

- xmsn line analysis necessary: line length is a significant fraction of $\lambda$ or longer

- xmsn line analysis not necessary: line length is a negligible fraction of $\lambda$

- electrical length (in degrees, radians, or wavelengths) vs. physical length (in mm, cm, m, etc.) of transmission lines

- characteristic impedance $Z_o$ is the ratio of $v$ to $i$ for forward-propagating wave and also for reflected wave; $Z_o$ is dependent on geometry and material properties

- concepts of “source” (or “generator”) and “load”

- concepts of “lumped elements” and “distributed” quantities ($R', L', G', C'$)

- lumped-element model ($R'\Delta z$, $L'\Delta z$, $G'\Delta z$, and $C'\Delta z$) – but you do not need to memorize the formulas in Table 2-1

- $L'$ and $C'$ are directly proportional to $\mu$ and $\varepsilon$, respectively, regardless of line type

- common types of transmission lines (basic geometries; see Fig. 2-4 in all editions)
  - coaxial cable
  - two-wire line
  - microstrip line
  - parallel-plate line

- transmission line (or telegrapher’s) equations:

  - time domain: $-\frac{\partial v(z,t)}{\partial z} = R'i(z,t) + L'\frac{\partial i(z,t)}{\partial z}$ and $-\frac{\partial i(z,t)}{\partial z} = G'v(z,t) + C'\frac{\partial v(z,t)}{\partial z}$

  - phasor domain: $-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$ and $-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$

- wave equation for phasor voltage and/or current along transmission lines

  - $\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$ and $\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$

- complex propagation constant ($\gamma$): $\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$
- solutions to the wave equation for both voltage and current:
  \[ V(z) = V_0^+ e^{-r z} + V_0^- e^{r z} = V_0^+ \left( e^{-r z} + \Gamma e^{r z} \right) \]
  \[ I(z) = I_0^+ e^{-r z} + I_0^- e^{r z} = \frac{V_0^+}{Z_o} \left( e^{-r z} - \Gamma e^{r z} \right) \]

- meanings of \( V_0^+ \) and \( V_0^- \) (and of \( I_0^+ \) and \( I_0^- \)); all four quantities can be complex
- how to find voltage (current) in time domain from phasor representation, & vice versa
- cosine representation of waves in time domain
- phasor representation of waves in frequency domain
- linearity and superposition (two or more waves traveling over the same transmission line simply add together)
- characteristic impedance \((Z_o)\)
  \[ I_0^+ = \frac{V_0^+}{Z_o} \quad \text{and} \quad I_0^- = -\frac{V_0^-}{Z_o} \]

- meanings of real and imaginary parts of propagation constant: \( \gamma = \alpha + j\beta \)
  \( \alpha \) = attenuation constant, measured in Np/m
  \( \beta \) = phase constant (or wavenumber), measured in rad/m

- permeability of all parts of almost all xmsn lines is free space value (i.e., \( \mu = \mu_o \))
  - typically, \( \mu \neq \mu_o \) for iron, cobalt, nickel and their compounds/ alloys
  - “nonmagnetic” means \( \mu = \mu_o \)

- definitions of and relationships between:
  - \( T \) = period (measured in s)
  - \( \lambda \) = wavelength (m)
  - \( f \) = frequency (Hz, or 1/s)
  - \( \omega \) = radian frequency (rad/s)
  - \( u_p \) = phase velocity (m/s)
  - \( \phi \) = phase (rad or deg)

- phase velocity of waves along transmission lines:
  - derivation using \( \frac{\partial}{\partial t} \cos(\alpha t - \beta z) = 0 \), or using
    \[ \cos(\alpha t - \beta z) = \cos[\alpha(t + \Delta t) - \beta(z + \Delta z)] = \text{constant}, \] where \( \Delta z \) is the distance traveled in time \( \Delta t \) by any single point on the cosine wave
  - \( u_p = \omega / \beta \) (i.e., \( u_p \) is positive and wave travels in \( +z \) direction) if one of the signs before \( \alpha t \) and \( \beta z \) is positive and the other is negative
  - \( u_p = -\omega / \beta \) (i.e., \( u_p \) is negative and wave travels in \( -z \) direction) if signs before \( \alpha t \) and \( \beta z \) are either both positive or both negative
  - \( u_p = \frac{c}{\sqrt{\varepsilon_r}} = (\text{v.f.})c = \frac{1}{\sqrt{\mu \varepsilon}}, \) where v.f. = velocity factor (v.f. = \( \varepsilon_r^{-1/2} \))

- for a lossless transmission line \((R' = 0 \text{ and } G' = 0)\),
  \[ \alpha = 0, \quad \beta = \omega \sqrt{L'C'} = \frac{\omega \sqrt{\varepsilon_r}}{c}, \quad Z_o = \frac{L'}{C'}, \quad L' = \frac{Z_o u_p}{u_p}, \quad \text{and} \quad C' = \frac{1}{Z_o u_p} \]
- for a low-loss transmission line \((R' \ll \omega L' \text{ and } G' \ll \omega C')\),
\[
\alpha \approx \frac{R'}{2Z_o} + \frac{G'Z_o}{2}, \quad \beta \approx \frac{\omega \sqrt{L'C'}}{c}, \quad Z_o \approx \frac{L'}{\sqrt{C'}}, \quad L' \approx Z_o, \quad \text{and } C' \approx \frac{1}{Z_o u_p},
\]
where \(Z_o\) is the nominal purely real value.

- units of sinusoidal quantity vs. units of argument (The magnitude of the cosine function represents a voltage or current and is measured in V or A, but the argument is in radians. That is, both \(\omega t\) and \(\beta Z\) are in radians.)

**Loaded transmission lines**

- In general, \(\frac{\hat{V}_{\text{tot}}(z_1)}{\hat{I}_{\text{tot}}(z_1)} \neq \frac{\hat{V}_{\text{tot}}(z_2)}{\hat{I}_{\text{tot}}(z_2)}\) where \(z_1\) and \(z_2\) are two different locations along line

- Ohm’s law must be satisfied at the load; that is, \(Z_L = \frac{\hat{V}_{\text{tot}}(0)}{\hat{I}_{\text{tot}}(0)} = \frac{\hat{V}_L}{\hat{I}_L}\)

- voltage reflection coefficient (\(\Gamma\))
  - complex-valued (be able to find magnitude \(|\Gamma|\) and phase \(\theta_f\)): \(\Gamma = |\Gamma| e^{i\theta_f}\)
  - definition: \(\Gamma = \frac{V_o^-}{V_o^+}\) usual formula: \(\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}\)
  - special cases (\(Z_L = 0, Z_L \to \infty, Z_L = Z_o, Z_L\) purely real or reactive)
  - phase of reflection coefficient usually expressed in the range \(-\pi \leq \theta_f \leq \pi\)

- standing waves
  - derivations of expressions for \(|\hat{V}(z)|\) and \(|\hat{I}(z)|\)
  - voltage magnitude maxima and minima \((V_{\text{max}}\) and \(V_{\text{min}}\))
  - current magnitude maxima and minima \((I_{\text{max}}\) and \(I_{\text{min}}\))
  - voltage maxima (current minima) occur where \(\cos(\theta_f + 2\beta z) = +1\)
  - voltage minima (current maxima) occur where \(\cos(\theta_f + 2\beta z) = -1\)
  - voltage and current maxima and minima repeat every \(\lambda/2\)
  - voltage maxima and current maxima separated by \(\lambda/4\)
  - voltage maxima and minima separated by \(\lambda/4\)
  - current maxima and minima separated by \(\lambda/4\)
  - voltage standing wave ratio: \(\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{V_{\text{max}}}{V_{\text{min}}}\)
  - \(\text{VSWR} = 1\) and \(V_{\text{min}} = V_{\text{max}}\) on a perfectly matched line \((Z_L = Z_o)\)
  - \(\text{VSWR} \to \infty\) and \(V_{\text{min}} = 0\) if load is 0, \(\infty\), or purely imaginary
  - \(\text{VSWR}, d_{\text{max}},\) and/or \(d_{\text{min}}\) on a mismatched line \((Z_L \neq Z_o)\) can be used to find \(Z_L\) (basis of operation of slotted line)
  - \(\text{VSWR}\) is often easier and/or cheaper to measure than \(\Gamma\)
  - \(\text{VSWR}\) can be the same for many different load impedances (because VSWR is a function only of \(|\Gamma|\), not of the full complex quantity)
  - For the coordinate system we are using, physically meaningful values of \(z\) are negative and of \(d\) (or \(l\)) are positive.
Transmission line impedance calculations
- electrical length (in degrees, radians, or wavelengths) vs. physical length (in mm, cm, m, etc.) of transmission lines
- input impedance of lossless line with load:
  - $Z_{in}(-l) = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$
  - alternate sometimes useful form: $Z_{in}(-l) = \frac{\tilde{V}(-l)}{I(-l)} = Z_o \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}$
  - open-circuit load ($Z_L = \pm \infty$): $Z_{in}(-l) = -jZ_o \cot(\beta l)$
  - short-circuit load ($Z_L = 0$): $Z_{in}(-l) = jZ_o \tan(\beta l)$
  - matched line ($Z_L = Z_o$): $Z_{in}(-l) = Z_o$
  - quarter-wave matching section ($l = \lambda/4$): $Z_{in}(-l) = \frac{Z_o^2}{Z_L}$
  - half-wave section ($l = n\lambda/2$): impedances repeat every $\lambda/2$ along line
  - purely reactive loads ($Z_L = jX$): $|\Gamma| = 1$; $Z_{in}$ is purely imaginary
  - “electrically short” lines (i.e., $l << \lambda$): $Z_{in} \approx Z_L$; this is the “circuit theory” limit; valid only if $Z_o >> |Z_L \tan(\beta l)|$ and $|Z_L ||Z_o \tan(\beta l)|$ (i.e., denominator of formula for $Z_{in}$ above is dominated by $Z_o$ and numerator is dominated by $Z_L$)
  - differences between $Z_o$ and $Z_{in}$ and $Z_L$
  - VSWR along line (especially along matching sections) changes at transitions, such as changes in $Z_o$ or locations where a device or component (such as an $R$, $L$, $C$, or antenna) is connected in parallel or in series with the line
  - lossy lines: $Z_{in}(-l) = Z_o \frac{Z_L + jZ_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}$, where $\gamma = \alpha + j\beta$ and $Z_o$ is complex

- calculation of $V_o^+$ and/or $\tilde{V}_{in}$, and the meanings of the two quantities
  $$V_o^+ = \tilde{V}_g \left( \frac{Z_{in}}{Z_g + Z_{in}} \right) \frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}$$
  $$\tilde{V}_{in} = \tilde{V}_g \left( \frac{Z_{in}}{Z_g + Z_{in}} \right)$$

  Note: if $Z_g = Z_o$, then $V_o^+ = \frac{1}{2} \tilde{V}_g e^{-j\beta l}$

- determination of total phasor or time-domain voltage or current at specific locations along line or their components [i.e., $\tilde{V}_{fwd}(z)$ and $\tilde{V}_{ref}(z)$ or $\tilde{I}_{fwd}(z)$ and $\tilde{I}_{ref}(z)$]
- cascaded line sections (one follows another) of different characteristic impedances
- equivalent impedance at junction of two or more line sections (i.e., “shunt” connections)
- input impedance when loads are located at different points along line

Impedance matching
- typical goals of impedance matching:
  - maximum power transfer
  - specific load impedance required for proper operation of source
  - to prevent reflections
  - combination of one or more of these
- quarter-wave matching sections (characteristic impedance \( Z_oQ \))
  - if \( Z_L \) is purely real and target matching impedance \( Z_{in} \) is purely real (usually the case), use a quarter-wave section with \( Z_{oQ} = \sqrt{Z_{in} Z_L} \)
  - for complex loads:
    - can connect stub at load to cancel load reactance (susceptance), if present
    - alternatively, can find location of voltage max or min, where \( Z_{in} \) is purely real:
      - voltage max: \( \theta_r - 2\beta d_{\text{max}} = 2\pi n \) and \( Z(-d_{\text{max}}) = Z_o \frac{1+|\Gamma|}{1-|\Gamma|} \)
      - voltage min: \( \theta_r - 2\beta d_{\text{min}} = (2n + 1)\pi \) and \( Z(-d_{\text{min}}) = Z_o \frac{1-|\Gamma|}{1+|\Gamma|} \)

if matching to main line of characteristic impedance \( Z_o \), use
\( Z_{oQ} = \sqrt{Z(-d_{\text{max}}) Z_o} \) or \( Z_{oQ} = \sqrt{Z(-d_{\text{min}}) Z_o} \)

Relevant homework, readings, and other resources:

HW: Assignments #1, #2, and #3
Reading: Assignments from Jan. 14 through Feb. 13 (except Sec. 2-8 in 5th ed. and Sec. 2-9 in 6th/7th eds. and the supplement “Impedance Matching Using Single Transmission Line Stubs”)
Web Links: NTIA Frequency Allocation Chart
RF Cafe’s Coaxial Cable Specifications Chart
Slotted Lines for Transmission Line Load Impedance Measurements
Examples of Microstrip Circuits
Matlab: none
Mathcad: none