ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Homework Assignment #3 – due via Moodle at 11:59 pm on Friday, Feb. 16, 2024

Instructions, notes, and hints:

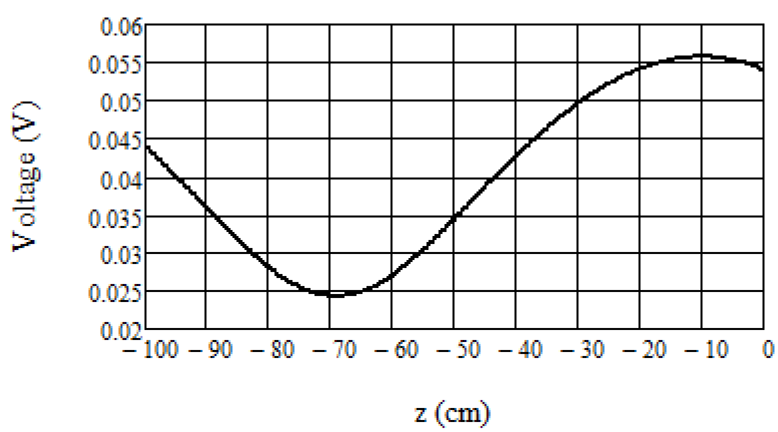
Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It might be necessary to use good engineering approximations or assumptions to solve all or part of these problems, especially if critical information is missing. In those cases, your answer might differ from the posted answer by a significant margin. If you justify any approximations that you make, you will be given full credit for such answers.

Note that the first six problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. Suppose that you have just formed a company that will market antennas designed for a new point-to-point wireless broadband delivery service. You cannot afford an expensive vector network analyzer, so instead you purchase an old slotted line to make impedance measurements. The slotted line is essentially lossless, has air insulation, and has a characteristic impedance of 50 Ω. You connect the line to your latest prototype antenna and obtain the plot shown below of the line voltage magnitude vs. distance from the load. Find the antenna's input impedance.



2. A 4.1-m section of 75 Ω coaxial line with polyethylene insulation is driven by a signal source as shown in the figure at the top of the next page. The generator (signal source) voltage expressed as a phasor is $V_g = 0.20 \angle 0°$ V rms, and the load impedance is $Z_L = 40 - j80$ Ω. Find the phasor input voltage V_{in} and the phasor load current I_L. Express both phasors in polar form. The operating frequency is 300 MHz.

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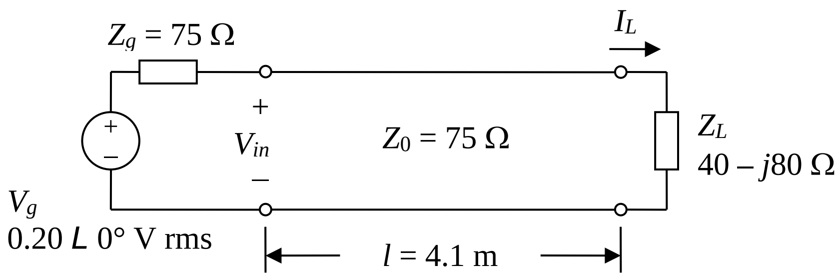
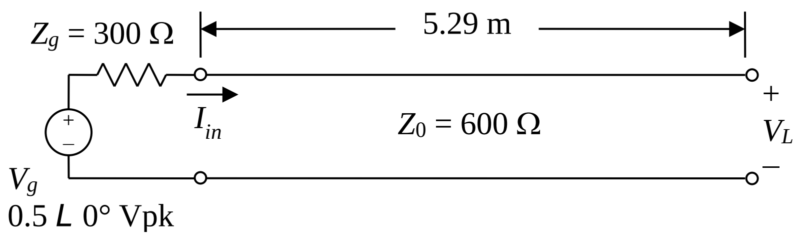
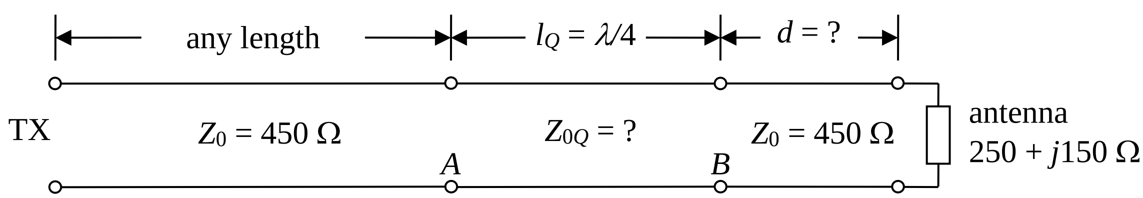


Diagram for Graded Prob. 2

3. Find the phasor representation (in polar form) of the load voltage V_L at the location of the open circuit at the far end of the transmission line stub shown below. Also find the phasor input current I_{in} of the stub. The operating frequency is 10 MHz, and the line is a parallel-wire type with air insulation.



4. Design a quarter-wave section to match an antenna with an impedance of 250 + j150 Ω at an operating frequency of 20 MHz to a 450 Ω parallel-wire air-insulated transmission line. The matching system is depicted schematically below. Find the distance d from the antenna to the load side of the quarter-wave section and the quarter-wave section's characteristic impedance Z_{0Q}. Also, find the required physical length l_Q of the quarter-wave section in meters. "TX" is a standard abbreviation for transmitter; it represents the signal source at the input end.



5. For the matching system considered in the previous problem, find:

- **a.** the VSWR along the line between the transmitter and the input of the quarter-wave section (point A).
- **b.** the VSWR along the quarter-wave section.
- **c.** the VSWR between the output of the quarter-wave section (point B) and the load.

6. Suppose that a capacitor with a reactance of −150 Ω at the operating frequency is inserted in series with the antenna considered in the previous two problems. The quarter-wave matching section is then redesigned. Find the new required distance d, quarter-wave section length l_Q, and characteristic impedance Z_{oQ}.

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***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Attempt to design a quarter-wave line section to achieve an impedance match between a load of $Z_L = j40\ \Omega$ and a transmission line with $Z_0 = 75\ \Omega$. The line and stub dimensions may be in wavelengths. Explain conceptually why this task is impossible.

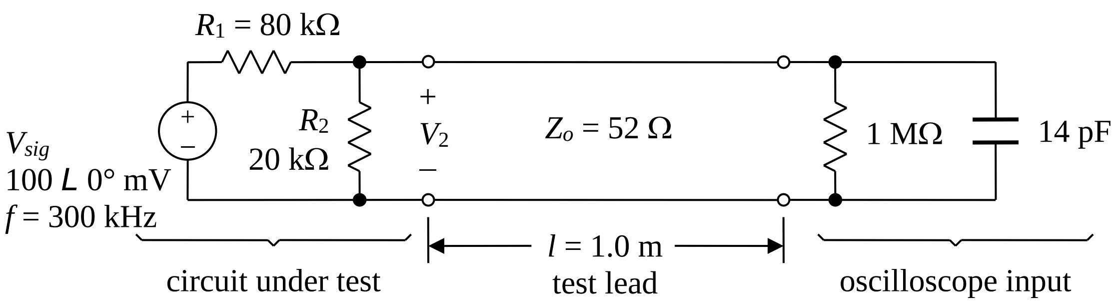
2. Use the expression given below for the input impedance of a loaded transmission line to show that any line with a purely reactive load (i.e., $Z_L = jX$, where X can be a capacitive or inductive reactance) must have an input impedance of zero at those locations z_{min} where voltage minima occur. You may use one of the expressions for d_{min} given in the textbook.

$$Z_{in} = Z_0 \frac{1+\Gamma e^{j2\beta z}}{1-\Gamma e^{j2\beta z}} = Z_0 \frac{1+|\Gamma| e^{j(\theta_r + 2\beta z)}}{1-|\Gamma| e^{j(\theta_r + 2\beta z)}}$$

3. Show that the coefficient V_0^+ used in the expression for the phasor voltage along a transmission line can be found using the alternative expression given below instead of Equation (2.82) in the textbook. One advantage of the expression below is that it gives the proper value for V_0^+ even if $Z_{in} = 0$ at the input end of the line. This can happen, for example, with a quarter-wave open-circuited transmission line stub.

$$V_0^+ = \tilde{V}_g \frac{Z_0}{\left(Z_0 + Z_g\right)e^{j\beta l} + \left(Z_0 - Z_g\right)\Gamma e^{-j\beta l}}$$

4. Suppose that you have been asked to measure the voltage across a voltage divider that operates at 300 kHz. The figure below depicts the signal source V_{sig} and the divider formed by R_1 and R_2. The circuit is electrically small (i.e., it is less than 0.01λ in size). To make the measurements, you are using an oscilloscope with a test lead made from a 1.0 m length of RG-58A coaxial cable ($Z_0 = 52\ \Omega$ with polyethylene dielectric). As shown in the figure, the oscilloscope's input port can be modeled as a resistance of 1 MΩ in a parallel with a capacitance of 14 pF. (These values are printed next to the input ports of most oscilloscopes, including the ones in the Bucknell ECE labs.) Find the phasor voltage V_2 (in polar form) that would be measured across the 20 kΩ resistor with the test lead connected across the resistor as shown. Compare your answer to the voltage that would appear across R_2 if the leads were not connected. Do the cable and oscilloscope load down the circuit significantly?



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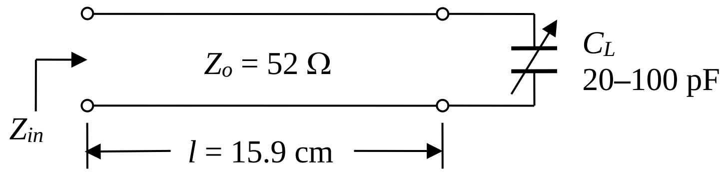
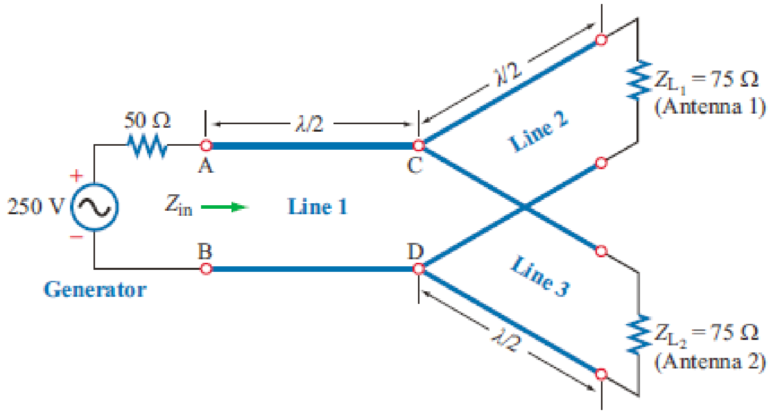
5. As shown below, a 20–100 pF variable capacitor is attached to the end of a 15.9 cm long section of RG-58A coaxial transmission line (Z_0 = 52 Ω with polyethylene insulation). The operating frequency is 90.5 MHz, which is the assigned frequency for Bucknell's FM radio station WVBU.

- **a.** Find the input impedance Z_{in} of the line for the minimum value of C_L (20 pF).
- **b.** Find the input impedance when C_L is at the high end of its range (100 pF).
- **c.** If possible, find the value of C_L (in pF) at which the line's input impedance Z_{in} is zero at 90.5 MHz.
- **d.** For the capacitance found in part c, find the input impedance of the line at 10 kHz.

The loaded stub considered in this problem is sometimes called a "wave trap." It is an effective way to keep a troublesome signal at a specific high frequency out of test equipment while minimizing the effect on low-frequency signals. The wave trap shown below would be placed in parallel with the test leads at the input port of a piece of test equipment (like an oscilloscope).



6. As shown below, two antennas are being fed by a single signal source. Line 1 has a characteristic impedance of 50 Ω, and lines 2 and 3 each have characteristic impedances of 75 Ω. Find the VSWR along each of the line sections (i.e., along lines 1, 2, and 3). Also find the input impedance Z_{in} of the whole antenna system at junction A-B.



Source: F. T. Ulaby and U. Ravaioli, *Fundamentals of Applied Electromagnetics*, 7th ed., Pearson Education, Inc., Upper Saddle River, NJ, 2015.