Homework Assignment #9 – due via Moodle at 11:59 pm on Friday, April 26, 2024

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

Some problems might require engineering approximations or assumptions to be applied to arrive at a solution, especially if critical information is missing. In those cases, your answer might differ significantly from the posted answer. If you justify any approximations you make, you will receive full credit for such answers. Unless otherwise indicated, you may use Matlab, Mathematica, or other software to make difficult or time-consuming calculations. If you do, include a print-out of the file or screen display that shows your work.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems. **Remember that you are responsible for understanding the concepts that are the focus of ungraded problems.** Some problems that might normally be included in the graded group were moved to the ungraded group for this assignment to avoid adding too much to your work load for the busy last week of the semester.

I strongly recommend that you work through the ungraded problems in this assignment. Some of the ungraded problems cover the last few topics of the semester that will not be covered by graded problems before the semester ends.

***Graded Problems*:**

1. The electric field component of a particular uniform plane wave can be described mathematically by the expression given below. The wave is propagating in a "nonmagnetic" medium, which means that $\mu = \mu_o$. The frequency of operation is 50 MHz. Find:

- **a.** a time-domain expression for the electric field in numerical form (i.e., with as many numerical values as possible substituted for variables and parameters).
- **b.** a time-domain expression for the magnetic field **H** in numerical form.
- **c.** the wavelength.
- **d.** the relative permittivity ε_r of the medium.

$$\tilde{\mathbf{E}} = \hat{\mathbf{y}}\,0.2e^{j0.3\pi}e^{-j2.1x} + \hat{\mathbf{z}}\,0.8e^{j0.3\pi}e^{-j2.1x}\ \ \mu\text{V/m}$$

2. A uniform plane wave with a frequency of 1.0 GHz is traveling through Teflon in the –*y* direction (i.e., –*y* is the direction of propagation). The electric field vector has a peak value of 0.1 mV/m and points in the direction 45° above the positive *x*-axis (i.e., bisecting the positive *x*-axis and the positive *z*-axis) during half of the wave period and in the opposite direction during the other half. The electric field is at its positive peak at all locations in the *y* = 2.0 cm plane at time *t* = 0. Find complete expressions for the *frequency*-domain (phasor) electric and magnetic fields.

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3. A uniform plane wave propagating in the +z-direction is partially represented by the mathematical expression shown below. The expression for the y-component (E_y) is missing. Find the mathematical form for E_y that results in the expression below describing a right-hand circularly polarized (RHCP) wave.

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}\,0.2e^{-j0.1\pi}e^{-j4.8z} + \hat{\mathbf{y}}\,E_y \ \text{ mV/m}$$

4. Plot the locus of the electric field vector as a function of time for the waves described by the phasor expressions given below. The "locus of the E-field vector" is the path followed by the tip of the vector (with the start of the arrow at the origin of the coordinate system) as time advances. This is the type of plot that has been produced for the in-class examples. Remember that the E-field vector is simply a mathematical representation. The actual E-field points in the same direction everywhere in any given plane at a particular moment in time. In each case listed below, identify the type of polarization. If it is circular or elliptical, specify the sense (left-hand or right-hand). If linear, specify the angle that the electric field makes with the y-axis.

a. $\tilde{\mathbf{E}} = \hat{\mathbf{y}}\,0.2e^{j0.3\pi}e^{-j2.1x} + \hat{\mathbf{z}}\,0.8e^{j0.3\pi}e^{-j2.1x}$ μV/m
b. $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\,5.0e^{j0.3\pi}e^{j0.22z} + \hat{\mathbf{y}}\,5.0e^{-j0.7\pi}e^{j0.22z}$ μV/m

5. The electric field component of a particular plane wave can be described mathematically by the expression given below right. Find the magnitude and direction of the electric field in free space at the locations given below left at times t_1 = 5.0 ns and t_2 = 10.0 ns. Briefly explain the interesting results that you obtain.

a. (x, y, z) = (1.0, 1.0, 1.0) m
b. (x, y, z) = (1.0, 2.0, 3.0) m
c. (x, y, z) = (1.0, 3.0, 5.0) m

$$\tilde{\mathbf{E}} = \hat{\mathbf{y}}\,0.2e^{j0.3\pi}e^{-j2.1x} + \hat{\mathbf{z}}\,0.8e^{j0.3\pi}e^{-j2.1x} \ \mu\text{V/m}$$

***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. The expression for the electric field in air of an elliptically polarized wave is given below. The frequency of operation is 100 MHz. A center-fed dipole is placed in a plane parallel to the yz-plane and is connected to a receiver. Find the orientation with respect to the y-axis (i.e., the angle that the dipole makes with the y-axis) that the dipole should have to maximize the received signal power. Find the signal power P_{rec} detected by the receiver. Assume that all antennas are 100% efficient and that there are no transmission line or mismatch losses.

$$\tilde{\mathbf{E}} = \hat{\mathbf{y}}5e^{-j0.2\pi}e^{-jkx} + \hat{\mathbf{z}}2e^{j0.3\pi}e^{-jkx} \ \mu\text{V/m}$$

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2. A transmitter that is part of a communication link generates an EM plane wave that propagates through air. It can be described mathematically by the expression shown below. The engineers analyzing the communication link define a coordinate system in which the transmitter is located at the origin and the receiving station is located 10 km away along the *x*-axis. Note that the E-field is expressed using spherical coordinates; if you wish, you may convert to Cartesian coordinates in the vicinity of the receiving station.

- **a.** Determine the polarization of the wave at the receiving site. If the polarization is circular or elliptical, specify the sense (left-hand or right-hand). If linear, specify the angle that the electric field makes relative to the $\hat{\boldsymbol{\theta}}$ direction.
- **b.** Find the power densities (Poynting vector magnitudes) of the θ and the ϕ-components of the electric field. Also find the power density of the full field.
- **c.** Find the corresponding phasor expression for the magnetic field in spherical coordinates.

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\varphi}}\, 7.0 e^{-j0.2\pi} \frac{e^{-j2.1R}}{R} + \hat{\boldsymbol{\theta}}\, 3.8 e^{j0.3\pi} \frac{e^{-j2.1R}}{R} \ \text{mV/m}$$

3. [adapted from Prob. 7.42 of Ulaby & Ravaioli, 7th ed.] A team of scientists is designing a radar to measure the thickness of an Antarctic ice shelf. If the echo due to the reflection from the ice-rock boundary is to be detectable, the thickness of the ice sheet should not exceed three skin depths. Assuming that $\varepsilon'_r = 3$ and $\varepsilon''_r = 10^{-2}$ for the ice shelf and that the maximum anticipated ice thickness in the area under exploration is 1.2 km, find the usable frequency range of the radar.

4. The constitutive parameters of soil are highly variable and depend on such attributes as moisture content, mineral content, and density. The antenna analysis software *EZNEC* uses the values $\varepsilon_r = 15$, $\mu_r = 1$, and $\sigma = 0.005$ S/m to represent "average" ground. Classify "average" ground as a good conductor, a quasi-conductor, or a low-loss dielectric at each of the following frequencies. For each case, also calculate the wavelength and the skin depth. List all of the results in tabular form at the end for convenience.

- **a.** 60 Hz (power line frequency in the US and many other countries)
- **b.** 60 kHz (operating frequency of WWVB, the NIST LF time standard station)
- **c.** 1070 kHz (operating frequency of WKOK)
- **d.** 90.5 MHz (operating frequency of WVBU)
- **e.** 2.44 GHz (middle of the Bluetooth frequency range)

5. The attenuation in air at sea level for signals at 30 GHz is approximately 0.14 dB/km. This frequency is close to one of many being considered for future 5G cellular systems. Suppose that the electric field component of a particular plane wave at this frequency propagating through air at sea level is expressed mathematically (and partially symbolically) as

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}} 37 e^{-\alpha z} e^{-j\beta z} \ \mu\text{V/m}.$$

- **a.** Find the attenuation constant α in Np/m.
- **b.** Find the phase constant β in rad/m.
- **c.** Find the numerical values of the power density at $z = 5.0$ m (close to the origin of the wave) and at $z = 5.0$ km. You may assume that air is a low-loss dielectric.

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6. Suppose that a transmitting station needs to produce a linearly polarized wave, but it also needs to be able to control the angle of polarization relative to the horizon. One solution is to mechanically rotate a linearly polarized antenna; however, the station is located in a cold climate, and ice accumulation could be a problem. An alternative is to mount two antennas with opposite circular polarization (CP) senses (i.e., one RHCP and the other LHCP) side-by-side and introduce a phase shift between them. There are circuits that can shift phase, and one could be inserted at the input of one of the CP antennas. The signals radiated by the two antennas can be represented by the mathematical expressions shown below. The distance from the pair of antennas to the receiving site is far enough that the radiated fields can be considered plane waves. The subscripts L and R indicate the polarization sense (left-hand and right-hand, of course). Use the superposition principle to show symbolically that the two signals add to produce a linearly polarized wave, and determine how the phase shift $\Delta\phi$ is related to the angle ζ that the E-field makes with the horizon. (The x-axis is parallel to the horizon.). A potentially helpful identity is also included below. Variable A is a real constant, and $k = 2\pi/\lambda$.

$$\tilde{\mathbf{E}}_L = \hat{\mathbf{x}}\, Ae^{-jkz} + \hat{\mathbf{y}}\, Ae^{j0.5\pi}e^{-jkz} \quad \text{and} \quad \tilde{\mathbf{E}}_R = \hat{\mathbf{x}}\, Ae^{j\Delta\phi}e^{-jkz} + \hat{\mathbf{y}}\, Ae^{-j0.5\pi}e^{j\Delta\phi}e^{-jkz}$$

$$1 + e^{j\Delta\phi} = e^{j0.5\Delta\phi}\left(e^{-j0.5\Delta\phi} + e^{j0.5\Delta\phi}\right) = e^{j0.5\Delta\phi}\left(e^{j0.5\Delta\phi} + e^{-j0.5\Delta\phi}\right) = 2e^{j0.5\Delta\phi}\cos\left(0.5\Delta\phi\right)$$

7. [adapted from Prob. 8.4 in the textbook (Ulaby & Ravaioli, 7th ed.)] A 200 MHz, LHCP plane wave with an E-field modulus of 5 V/m is normally incident in air upon a dielectric medium with $\varepsilon_r = 4$. The dielectric occupies the region defined by $z > 0$.

- **a.** Find a mathematical expression for the E-field phasor of the incident wave, given that the field is at a positive maximum at $z = 0$ and $t = 0$.
- **b.** Calculate the reflection and transmission coefficients applicable at the boundary between air and the dielectric medium.
- **c.** Find mathematical expressions for the E-field phasors of the reflected wave and the transmitted wave.
- **d.** Determine the percentages of the incident average power reflected by the boundary and transmitted into the dielectric.
- **e.** Determine the polarizations of the reflected and transmitted waves.