Please review the “Exam Policies” section of the Exams page at the course web site. Also please note the following two changes from policies used in the past:

1. You will be allowed to use three 8.5 × 11-inch two-sided handwritten help sheets. There are no restrictions on the material you may place on the help sheets, except that no photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam, please notify the instructor.

2. All help sheets will be collected at the end of the exam but will be returned to you later.

The final exam will have a take-home portion that will consist of one question. It will address Course Outcome #9, “Articulate the primary points of debate concerning a contemporary issue related to the use of the electromagnetic spectrum.” You will be assigned a reading on a contemporary issue and will be asked to write a short reflection on the reading.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. You should be familiar with the topics on the review sheets for the previous exams in addition to those listed below.

Short transmission lines as series inductances or parallel capacitances
- if \( Z_o \gg |Z_L \tan(\beta l)| \) and \( \tan(\beta l) \approx \beta l \), then \( Z_o(-l) \approx Z_L + jZ_o \beta l \), and

\[
L_{\text{eff}} = \frac{Z_o \beta l}{\omega} = \frac{Z_o \beta l}{2 \pi f \frac{\lambda}{\omega}} = \frac{Z_o \beta l}{\nu_p}. \quad \text{Note that} \quad \frac{Z_o}{\nu_p} = \sqrt{\frac{L'}{C'}} = L'.
\]

where \( L' \) is the inductance per unit length along the line.
Significance: A short high-impedance line can cancel a series capacitive reactance.

- if \( Y_o \gg |Y_L \tan(\beta l)| \) and \( \tan(\beta l) \approx \beta l \), then \( Y_o(-l) \approx Y_L + jY_o \beta l \), and

\[
C_{\text{eff}} = \frac{Y_o \beta l}{\omega} = \frac{Y_o \beta l}{\omega Z_o} = \frac{1}{2 \pi f Z_o} \left( \frac{2 \pi}{\lambda} \right) = \frac{l}{Z_o \nu_p}. \quad \text{Note that} \quad \frac{1}{Z_o \nu_p} = \sqrt{\frac{C'}{L'}} = C'.
\]

where \( C' \) is the capacitance per unit length along the line.
Significance: A short low-impedance (high-admittance) line can cancel a shunt inductive reactance.

- short line sections are not effective at canceling series inductance or shunt capacitance without changing load resistance significantly

Single-stub impedance matching
- shunt-stub matching (more common w/coaxial and parallel-wire lines, also w/microstrip)
  - place stub at any location where \( \text{Re}\{Y_{in}\} = Y_o = 1/Z_o \)
  - two possible locations every \( \lambda/2 \): \( I_{\text{main}} = \frac{\lambda}{4\pi} \left[ \theta_e + \cos^{-1}(-|\Gamma_L|) \right] \), where

\[
\Gamma_L = |\Gamma_L| e^{j\theta_e}.
\]
  - can also use any location \( n\lambda/2 \) away from those given by formula (\( n = \text{integer} \))
\( B_{in} = \pm \frac{2|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}} Y_o \); make \( B_{stub} = -B_{in} \) (can use \( L \) or \( C \) instead of stub)

- series-stub matching (common w/microstrip and lumped \( C \) or \( L \))
  - place stub or lumped element at any location where \( \text{Re}\{Z_{in}\} = Z_0 \)
  - two possible locations every \( \lambda/2 \): \( l_{main} = \frac{\lambda}{4\pi} \left[ \theta_o \pm \cos^{-1}(|\Gamma_L|) \right] \)
  - \( X_{in} = \mp \frac{2|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}} Z_0 \); make \( X_{stub} = -X_{in} \) (can use \( L \) or \( C \) instead of stub)

- short- and open-circuited stubs to cancel input susceptance \( B_{in} \) or input reactance \( X_{in} \)
  - short-circuited stub: \( l_{scstub}^{sc} = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{X_{stub}}{Z_o}\right) = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{-Y_o}{B_{stub}}\right) \)
  - open-circuited stub: \( l_{ocstub}^{oc} = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{-Z_o}{X_{stub}}\right) = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{-Y_o}{B_{stub}}\right) \)

- add \( \lambda/2 \) if formula for \( l_{main} \) or \( l_{stub} \) gives a negative length

Smith chart
- based on the \( \Gamma \) plane (plot of \( \Gamma_i = \text{Im}\{\Gamma\} \) vs. \( \Gamma_r = \text{Re}\{\Gamma\} \))
- uses include:
  - alternative method to calculate \( l_{main} \) and \( l_{stub} \) in stub-matching problems; can also be used for double and triple-stub matching problems
  - single and multi-stage L network design
  - representation of impedance vs. frequency data
  - filter design
  - gaining insight when formulas do not provide it
  - many other uses!
- concept of normalizing impedance (or admittance)
- switch between equivalent impedance and admittance values by moving to diametrically opposite point on chart (only necessary if \( g \) and \( b \)-circles are not provided)
- constant-\( |\Gamma| \) (a.k.a. constant-VSWR) circles:
  - centered at origin
  - motion along corresponds to transmission line length changes
  - angular travel around circle is twice electrical length of line (2\( \beta l \))
  - motion away from load is always clockwise (for \( Z \) chart or for \( Y \) chart), because \( \Gamma(x) = \Gamma(-l) = \Gamma_l e^{j2\beta l} = \Gamma_l e^{-j2\beta l} \)
- constant-\( r \) circles:
  - centered at the point \( (\Gamma_r, \Gamma_i) = (r/r+1, 0) \), radius = 1/(\( r+1 \))
  - motion along corresponds to change in series \( x \) (e.g., adding series element)
- constant-\( x \) circles:
  - centered at \( (1, 1/x) \), radius = 1/|\( x \)|
  - motion along corresponds to change in series \( r \) (rarely used)
- constant-\( g \) circles:
  - centered at \( (-g/g+1, 0) \), radius = 1/(\( g+1 \))
  - motion along corresponds to change in parallel \( b \) (e.g., adding shunt element)
- constant-\( b \) circles:
  - centered at \( (-1, -1/b) \), radius = 1/|\( b \)|
  - motion along corresponds to change in parallel \( g \) (rarely used)
- convert “impedance” chart to “admittance” chart by turning chart upside-down
- inductance always occupies upper half of Smith chart (positive x, negative b)
- capacitance always occupies lower half of Smith chart (negative x, positive b)

Antenna analysis (determination of radiation fields from current distributions)
- all time-varying currents act as sources and potentially can radiate EM waves
- position vectors:
  - r (unprimed) points to observation point
  - r’ (primed) points to location on antenna
  - spherical coordinates: \( \mathbf{r} = \hat{r}r \) (direction depends on \( \theta \) and \( \phi \))
  - cylindrical coordinates: \( \mathbf{r} = \hat{\rho}\rho + \hat{z}z \) (direction depends on \( \phi \))
  - rectangular coordinates: \( \mathbf{r} = \hat{x}x + \hat{y}y + \hat{z}z \) (direction fully defined by \( x, y, z \))
- far fields of short dipole (triangular current distrib.) of length \( h \) w/peak input current \( I_o \):
  \[
  \mathbf{E} = \hat{\theta} \frac{j k \eta \tilde{I}_o h e^{-jkr}}{8\pi r} \sin \theta, \quad \mathbf{H} = \hat{\phi} \frac{j k I_o h e^{-jkr}}{8\pi r} \sin \theta
  \]
- far fields of Hertzian dipole (uniform distrib.) of length \( h \) w/peak input current \( I_o \):
  \[
  \mathbf{E} = \hat{\theta} \frac{j k \eta \tilde{I}_o h e^{-jkr}}{4\pi r} \sin \theta, \quad \mathbf{H} = \hat{\phi} \frac{j k I_o h e^{-jkr}}{4\pi r} \sin \theta
  \]
- far fields of half-wave dipole w/peak input current \( I_m \):
  \[
  \mathbf{E} = \hat{\theta} \frac{j \eta \tilde{I}_m e^{-jkr} \cos(0.5\pi \cos \theta)}{2\pi r} \sin \theta, \quad \mathbf{H} = \hat{\phi} \frac{j \tilde{I}_m e^{-jkr} \cos(0.5\pi \cos \theta)}{2\pi r} \sin \theta
  \]

Common characteristics of far fields for any antenna:
- the \( e^{-jkr} \) factor – implies spherical waves
- prop. in \( \hat{r} \) direction (if antenna is centered at origin) – also implies spherical waves
- speed of propagation is \( \frac{1}{\sqrt{\mu\varepsilon}} \) (speed of light in medium)
- electric and magnetic fields are proportional to input current and/or voltage
- \( \mathbf{E} \perp \mathbf{H} \), \( \mathbf{E} \perp \mathbf{P}_D \), and \( \mathbf{H} \perp \mathbf{P}_D \) (\( \mathbf{P}_D \) = time average Poynting vector; sometime the symbol \( S_m \), is used, as in Ulaby’s text); i.e., transverse EM (TEM wave)
- electric and magnetic fields are in phase (if waves travel through lossless or slightly lossy medium)
- \( |\mathbf{E}| = \eta |\mathbf{H}| \)

Common characteristics of far fields for dipoles oriented along z-axis:
- all of the above, plus
- electric field is \( \theta \)-directed (\( \theta \)-polarized); magnetic field is \( \phi \)-directed

Time-average Poynting vector
- definition \( \mathbf{P}_D = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*) \); many books prefer the variable \( \mathbf{S} \) or \( \mathbf{S}_{av} \)
- not a time-varying quantity; also not a phasor
- gives the time-average power density per unit area of an EM wave (unit is W/m²)
- points in the direction of power flow and propagation of phase fronts

Radiation pattern (sometimes also referred to as a “power pattern”)
- plot of normalized \( |\mathbf{P}_D| \) vs. \( \theta \) and/or \( \phi \) (know what “normalization” means)
- normalized power pattern: \( F(\theta, \phi) = |\mathbf{P}_D(\theta, \phi)| / P_{D,max} \)
- interpretation of radiation pattern plot

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Directivity
- calculation of input power to antenna using \( P_D \) (integration over all \( \theta \) and \( \phi \))
- concept of isotropic radiator
  - hypothetical antenna that radiates with equal intensity in all directions
  - no specified vector direction of fields; i.e., no polarization (not realistic)
  - Poynting vector of isotropic radiator:
    \[ P_{D,i} = \hat{r} \frac{P_{in}}{4\pi r^2} \]
- definition of directivity: \( D = \frac{P_{D,max}}{P_{D,i}} \). Some textbooks use \( D = \frac{S_{max}}{S_{iso}} \).
- directivity calculated from power pattern
  \[ D = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta \, d\theta \, d\phi} \]
- interpretation of directivity as inversely proportional to “volume” inside \( F(\theta, \phi) \)
  (remember that \( \max\{F(\theta, \phi)\} = 1 \))
- dBi = directivity (and gain) measured in decibels referenced to isotropic radiator
- directivities of short dipole and Hertzian dipole are 1.5 (1.76 dBi), because normalized power patterns are the same
- directivity of half-wave dipole is 1.64 (2.15 dBi)

Gain \( (G) \) and efficiency \( (\xi) \)
- sometimes the symbol \( \eta \) is used for efficiency; take care not to confuse efficiency with intrinsic impedance in this case (pay attention to context)
- \( G = \xi D \)
- \( \xi = \frac{R_{rad}}{R_{rad} + R_{loss}} \), if \( R_{rad} \) and \( R_{loss} \) are modeled in series with each other
- loss resistance usually represents finite conductivity of antenna structure and/or ground beneath it
- calculation of power density (Poynting vector magnitude) for given distance, gain (or efficiency and directivity), and input power:
  \[ P_{D,max} = \frac{P_{in} G}{4\pi r^2} = \frac{P_{in} \xi D}{4\pi r^2} \]
- alternate definition of gain (if some of \( P_{in} \) dissipates as heat) or directivity (if all of \( P_{in} \) is radiated and \( \xi = 1 \))
  \[ G = \frac{P_{D,max}}{P_{D,i}} = \frac{4\pi r^2 P_{D,max}}{P_{in}} \text{ or } G = \frac{S_{max}}{S_{iso}} = \frac{4\pi r^2 S_{max}}{P_{in}} \]

Radiation resistance [Note: This topic will not be covered on the final exam, but the notes below are offered to provide context and edification.]
- real part of equivalent input impedance of antenna that accounts for radiated power
- definition: \( R_{rad} = \frac{2P_{in}}{|I_{in}|} \), where \( I_{in} \) represents peak (not rms) input current. Note that \( I_{in} \) might not be the peak value of the current distribution along the antenna; it is simply the current at the input terminals.
- calculation of input power (equal to radiated power if no losses)

\[ P_{in} = P_{rad} = \int_{0}^{2\pi} \int_{0}^{\pi} P_D(\theta, \phi) \cdot \hat{r} r^2 \sin \theta \, d\theta \, d\phi \]

- short dipole in free space: \( R_{rad} = 20\pi^2 \left( \frac{h}{\lambda} \right)^2 \) (approx. 10% accuracy for \( h \approx 0.25\lambda \))

- Hertzian dipole in free space: \( R_{rad} = 80\pi^2 \left( \frac{h}{\lambda} \right)^2 \)

- half-wave dipole in free space: \( R_{rad} \approx 73 \, \Omega \)

Input reactance [Not covered on final exam]

- open-circuit-terminated transmission line model for dipole antennas
- short-circuit-terminated transmission line model for loop antennas
- short and Hertzian dipoles have extremely large capacitive reactance
- small loops have extremely large inductive reactance
- half-wave (nominally) dipoles
  - capacitive if shorter than first resonant length
  - inductive if longer than first resonant length

Yagi-Uda arrays

- multiple dipoles in parallel; lengths are nominally half-wave
- one element is driven, others are parasitically coupled
- reflector is slightly longer than driven element and has inductive self-impedance
- directors are slightly shorter than driven element and have capacitive self-impedances
- radiation resistance of driven element might not be 73 \( \Omega \) due to mutual coupling with other elements; could require matching network

Computational methods like EZNEC are required to find actual current distributions to high accuracy along real antennas.

Relevant coursework, textbook sections, and supplemental material:

Homework: #6
Labs: #5
Textbook: Secs. 4.9-4.14, 5.1-5.4, 5.6, 5.8, 5.12, 5.13, 7.23
Supplements: Classic Smith chart
Impedance chart
“Impedance Matching Using Single Transmission Line Stubs”
“Radiation Power and Directivity of Antennas”
“Radiation Resistance, Efficiency, and Gain of Antennas”
“Loss Resistance Calculations for Arbitrary Current Distributions”