Homework Assignment #4 – due via Moodle at 11:59 pm on Wednesday, Mar. 1, 2023

***Instructions, notes, and hints*:**

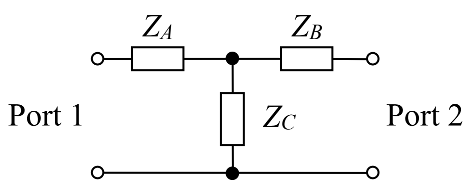
Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

It is your responsibility to review the solutions when they are posted (including those for ungraded problems) and to understand and rectify any conceptual errors that you might have. You may contact me at any time for assistance.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems. One graded problem will be randomly selected for detailed evaluation; the others will be evaluated using a coarse rubric.

***Graded Problems*:**

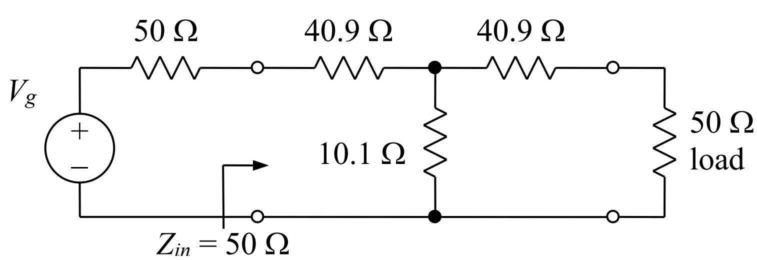
1. Show that the S-parameters S_{11} and S_{21} for the network depicted below are expressed in symbolic form by the formulas next to the diagram. The reference characteristic impedance is Z_0. Assume that the connecting wires have negligible length.



$$S_{11} = \frac{(Z_A - Z_0)(Z_B + Z_C + Z_0) + Z_C(Z_B + Z_0)}{(Z_A + Z_0)(Z_B + Z_C + Z_0) + Z_C(Z_B + Z_0)}$$

$$S_{21} = \frac{2Z_0 Z_C}{(Z_A + Z_0)(Z_B + Z_C + Z_0) + Z_C(Z_B + Z_0)}$$

2. Use the results of the previous problem to find the numerical values of S_{11} and S_{21} for the 20 dB T-network attenuator shown below. Express the magnitude of S_{21} in dB. The reference impedance is Z_0 = 50 Ω. The attenuator circuit consists of the 40.9 Ω and 10.1 Ω precision resistors shown in the diagram; the other components represent a signal source and a load. Assume that all resistors are ideal. Note that in this case the S-parameters will be real and independent of frequency. Briefly explain the implications of your results; that is, discuss why they make physical sense.



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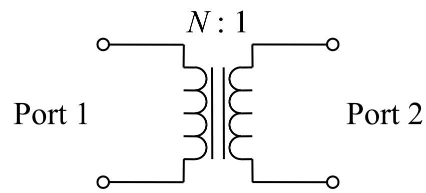
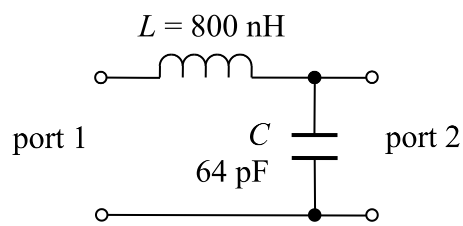
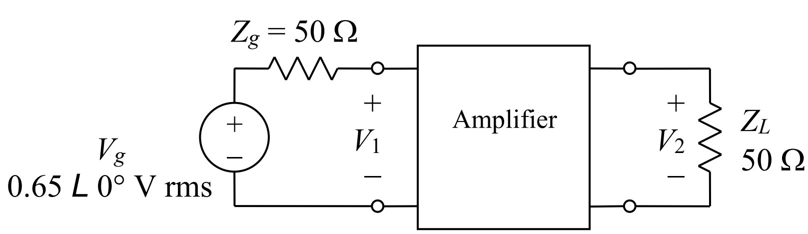
3. Find the S parameters S_{11} and S_{21} in symbolic form for an ideal lossless transformer with a turns ratio of N:1 (with N turns on the port 1 side) as shown below. The reference characteristic impedance is Z_0. Assume that the connecting wires have negligible length.



4. Find the numerical values of all four S-parameters (S_{11}, S_{21}, S_{12}, and S_{22}) for the L-shaped impedance matching network shown below at an operating frequency of 20 MHz (the design frequency for the matching network). The reference impedance is $Z_0 = 50\ \Omega$. The network is designed to match the impedance connected to port 2 to a 50 Ω signal source connected to port 1. Use one or more of the S-parameter values to determine the value of the expected load impedance for which the network was designed.



5. The S-parameters of an amplifier operating at 4.2 GHz are measured and found to have the values listed below the diagram. The reference impedance is $Z_0 = 50\ \Omega$. As shown in the diagram, a signal source is applied to the input of the amplifier with the indicated phasor source voltage, and a load is connected to the output. Find the power in milliwatts delivered to the load. *Hint*: The available power is 2.1 mW.



$S_{11} = 0.16\angle -20°$
$S_{21} = 8.7\angle -12°$
$S_{12} = 0.051\angle -140°$
$S_{22} = 0.22\angle 119°$

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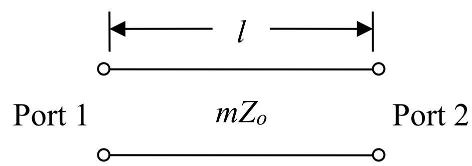
Ungraded Problems:

The following problem will not be graded. However, you should attempt to solve it on your own and then check the solution. Try not to give up too quickly if you struggle to solve it. Move on to a different problem and then come back to the difficult one after a few hours.

1. Show that the S parameters S_{11} and S_{22} of a transmission line of length l with characteristic impedance mZ_0, where m is a real multiplying factor (that's why it's "m") greater than or equal to one, are given by

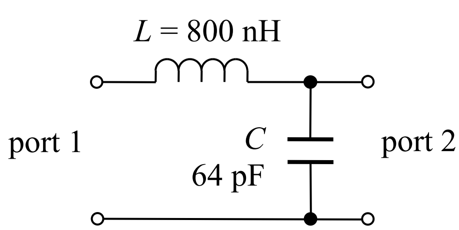
$$S_{11} = S_{22} = \frac{j\left(m^2 - 1\right)\tan\beta l}{2m + j\left(m^2 + 1\right)\tan\beta l}$$

The reference impedance is Z_0. A diagram of the line section is shown below:



Note that $S_{11} = S_{22} = 0$ when $m = 1$.

2. Find the numerical values of all four Z-parameters (Z_{11}, Z_{21}, Z_{12}, and Z_{22}) for the impedance matching network shown below at an operating frequency of 20 MHz. The network is designed to match the impedance connected to port 2 to a 50 Ω signal source connected to port 1. Use the conversion formula given below (adapted from D. Pozar, *Microwave Engineering*, 3rd ed., John Wiley & Sons, Inc., 2005, Sec. 4.3) to find the S-parameters, and compare their values to the ones found in the corresponding graded problem above. In the conversion formula, $[Z]$ is the Z-parameter matrix, and $[Z_0]$ is a diagonal matrix in which all of the entries on the main diagonal are Z_0 (50 Ω).



$$[S] = \left([Z] + [Z_0]\right)^{-1}\left([Z] - [Z_0]\right)$$