Instructions, notes, and hints:

You may make reasonable assumptions and approximations in order to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work. Use SI units, unless others are specified. If you use Matlab to make difficult or time-consuming calculations, include a commented print-out of the file or screen display that shows your work.

Assignment:

1. Show that the frequencies $f_C$ and $f_L$ at which each circuit shown below exhibits the indicated phase shift is given by the expressions above their respective diagrams. Also show that the indicated relationship between $|v_o|$ and $|v_i|$ applies at those frequencies for both circuits. The resistor $R$ (across which $v_o$ is labeled) in each circuit represents the input impedance of the local oscillator port of a mixer. These circuits could be used to implement $\pm 45^\circ$ phase shifts in an image-rejection mixer, although a high-value attenuator pad would probably need to be inserted before the mixer port to make sure its input impedance is very close to $Z_o$. Of course, these circuits would perform exactly as designed only at a single frequency (or reasonably well over a narrow range of frequencies).

\[
\begin{align*}
  f_C &= \frac{1}{2\pi RC} \\
  |v_o| &= \frac{1}{\sqrt{2}} \text{ at } f_C \text{ or } f_L \\
  f_L &= \frac{R}{2\pi L}
\end{align*}
\]

\[\begin{array}{c}
  v_{in} \\
  - \\
\end{array} + \quad C \quad + \\
\] \[\begin{array}{c}
  + \\
  v_{in} \\
  - \\
\end{array}
\]

phase shift = $+45^\circ$ @ $\omega_o$

\[\begin{array}{c}
  L \\
  \quad \\
\end{array} + \quad R = Z_o \quad + \\
\] \[\begin{array}{c}
  + \\
  v_{in} \\
  - \\
\end{array} + \quad R = Z_o \quad + \\
\] \[\begin{array}{c}
  + \\
  v_o \\
  - \\
\end{array}
\]

2. Find the S parameters in symbolic form for the series impedance shown below. The reference characteristic impedance is $Z_o$. Assume that the connecting wires have negligible length. You may take advantage of symmetry to minimize the work required.

\[
\begin{array}{c}
  Z_S \\
  \quad \\
\end{array} \\
\] \[\begin{array}{c}
  Port 1 \\
  - \\
\end{array} \quad \quad \quad Port 2 \\
\]

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3. Find the S parameters $S_{11}$ and $S_{21}$ in symbolic form in terms of $Z_A$, $Z_B$, and $Z_C$ of the generalized pi-network shown below. Show the details of your solution for each parameter. You may take advantage of symmetry to minimize the work required.

![Generalized Pi-Network Diagram]

4. Find the S parameters $S_{11}$ and $S_{21}$ in symbolic form for an ideal lossless transformer with a turns ratio of $N:1$ (with $N$ turns on the port 1 side) as shown below. The reference characteristic impedance is $Z_o$. Assume that the connecting wires have negligible length.

![Ideal Lossless Transformer Diagram]

5. Show that the S parameters $S_{11}$ and $S_{21}$ of a transmission line of length $l$ with characteristic impedance $mZ_o$, where $m$ is a real multiplying factor (that’s why it’s “$m$”) greater than or equal to one and $Z_o$ is the reference impedance for the S parameters, are given by

$$S_{11} = \frac{j(m^2 - 1)\tan \beta l}{2m + j(m^2 + 1)\tan \beta l} \quad \text{and} \quad S_{21} = \frac{4me^{-j\beta l}}{(1 + m)^2 - (1 - m)^2 e^{-j2\beta l}}.$$ 

A diagram of the line section is shown below:

![Transmission Line Diagram]

It might be helpful to recall from ELEC 390 that the total phasor voltage $V(x)$ along a transmission line is given by (using J. F. White’s notation)

$$V(x) = V_{\text{fwd}}(x) + V_{\text{ref}}(x) = V_l e^{-j\beta x} + V_r e^{j\beta x} = V_l (e^{-j\beta x} + \Gamma_L e^{j\beta x}),$$

where $x = 0$ is the location of the load and where $\Gamma_L$ is the reflection coefficient at the load, which in this case is given by

$$\Gamma_L = \frac{Z_L - mZ_o}{Z_L + mZ_o}.$$ 

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because the characteristic impedance of the line is $mZ_o$, not $Z_o$. Also recall that the coefficient $V_I$ can be found from

$$V_I = V_g \frac{Z_{in}}{Z_g + Z_{in}} \cdot \frac{1}{e^{i\beta l} + \Gamma e^{-i\beta l}} ,$$

where $Z_{in}$ is the input impedance seen at the signal source end of the line and $Z_g$ is the output impedance of the signal source (which should be set equal to $Z_o$ in this case). A useful expression for the input impedance of a loaded line section at a distance $l$ from the load for this case is

$$Z_{in}(-l) = mZ_o \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} .$$

6. Find the overall standard noise figure of the system shown below. The passband of the first filter (after the transmission line) is 9.5-10.5 GHz, and that of the IF filter is 1.10-1.11 GHz. The local oscillator tunes 8.9-9.2 GHz. There is no significant selectivity (i.e., no other narrow filters) beyond the IF amplifier. All stages of the receiver operate at a temperature of 290 K. The variable $L_c$ represents the conversion loss of the mixer. It indicates that the power level of the output IF signal is $1/L_c$ times the power level of the input RF signal. The indicated noise figures in the diagram are standard.

7. Repeat the previous problem with the positions of the transmission line and the LNA interchanged. Compare the results of the previous problem and this one, and comment.

8. You have been asked to design a receiver for the 2.4-2.5 GHz frequency range that is to have a system noise figure no greater than 3.0 dB. A block diagram of the receiver’s front end is shown below. The input band-pass filter, IF filter, mixer, and IF amplifier circuits have already been selected, and they have the indicated losses, noise figures, and gains. (The mixer uses active devices, so it has gain.) The transistor selected for the LNA leads to a noise figure of 0.5 dB if it is biased properly. The IF filter sets the overall bandwidth of the receiver to 20 MHz, and the circuitry following the IF amplifiers contributes negligible noise. The transmission line between the antenna and the receiver’s input port has negligible loss. Find the minimum required gain of the LNA that will allow you to achieve the desired system noise figure. The indicated noise figures in the diagram are standard.