ENGR 695 Advanced Topics in Engineering Mathematics Fall 2023

Lecture Outline for Wednesday, Aug. 23

1. Linear systems, linear algebra, and matrix equations [review]

    a. Multiple linear equations
    b. Formulation: knowns and unknowns
    c. Basic question: solutions…when and how?

2. Notation and conventions [review]

    a. Scalars, vectors, and matrices
    b. Size and shape (rows, columns)
    c. Typography and notational items

3. Rules of linear algebra (matrix algebra) [review]

    a. Addition, subtraction: size & shape matter; order does not
    b. Multiplication
        i. Many options (s-v, s-m, v-v, v-m, m-m; s = scalar, v = vector, m = matrix)
        ii. Size & shape matter
        iii. Possible loss of commutativity: direction and order matter
    c. Division
        i. Not always possible or even defined
        ii. When defined, it is through multiplication by an inverse

4. Basic computations in linear algebra: $A\mathbf{x} = \mathbf{b}$ [review & preview]

    a. A, $\mathbf{x}$ given: geometric transformations (images, outputs from inputs)
    b. A, $\mathbf{b}$ given: system solution (inputs from outputs)
    c. Size & shape matter in defining the solution
    d. Geometrical interpretations
    e. Unit vectors, projections, components, dot products

(*continued on next page*)

Example Problems in Matrix Arithmetic

Use the following vector and matrix definitions:

$$\mathbf{c} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Compute the following quantities. You are encouraged to use *Matlab* if you have access to it.

1. $\mathbf{c}^T\mathbf{c}$, where superscript T indicates the matrix transpose operation. What attribute of $\mathbf{c}$ does this operation yield?

2. $\mathbf{b}^T\mathbf{c}$ What is the geometrical interpretation of this operation?

3. $\mathbf{b}\mathbf{c}^T$

4. $A\mathbf{b}$

5. AB

6. $I\mathbf{c}$ What is I usually called? What is the key property of I?

7. Without resorting to computation, what are the results of the operations AI and IA?

8. $I + \mathbf{b}\mathbf{c}^T$

9. A^T

10. What would be required of a matrix D so that D^2 (which equals DD), D^TD, and DD^T all yield the same result? Does the matrix A meet this requirement? Why or why not?

Adapted from lecture outline by Prof. James Maneval, Fall 2019