ENGR 695 Advanced Topics in Engineering Mathematics Fall 2023

Lecture Outline for Wednesday, Sept. 27

1. Ordinary differential equations (ODEs): Scope and coverage

   a. Important special cases
   b. 2nd order linear equations are some of the most important equations of mathematical physics and will be our focus
   c. Distinction between and IVP and BVP (at how many points are the dependent variable and/or its derivatives defined?)
   d. Linear diff eqs that are IVPs have a unique solution
      i. IVP: $a_n(x)\dfrac{d^n y}{dx^n}(x)+a_{n-1}(x)\dfrac{d^{n-1}y}{dx^{n-1}}+\cdots+a_0(x)y=g(x)$
         $y(x_0)=y_0,\quad y'(x_0)=y_1,\quad \ldots \quad y^{(n-1)}(x_0)=y_{n-1}$ (defined at $x = x_0$)
      ii. BVP: dependent variable and/or derivatives defined at two or more points
      iii. BVPs can have no solution, a unique solution, or many solutions
   e. BVPs will be the main emphasis
   f. Special class of BVPs: eigenvalue problems

2. Solution strategies

   a. Identify class of ODE
   b. Applicable solution structure
   c. Corresponding solution method
   d. Starting point: homogenous solutions

3. Solvable class #1: Constant coefficients

   a. Chapter 2 covers 1st order with constant coefficients (skim)
   b. Section 3.3 covers 2nd order with constant coefficients
   c. Proposed solution and consequence

   Cases and solution forms (special case: Fourier equation: $y''+a^2y=0$ or $y''-a^2y=0$)

4. Solvable class #2: Variable coefficients

   a. Some general forms considered:
      i. Cauchy-Euler equation: $x^2y''+xy'+y=0$
      ii. Bessel equation: $x^2y''+xy'+\left(x^2-n^2\right)y=0$
      iii. Legendre equation: $\left(1-x^2\right)y''-2xy'+n\left(n+1\right)y=0$
      iv. others
   b. Proposed solution and consequence
   c. Cases and solution forms