ENGR 695 Advanced Topics in Engineering Mathematics Fall 2023

Lecture Outline for Monday, Aug. 28

1. Route to finding solutions (implicit inverse computation)

   a. Process: for augmented matrix, reduce (transform) a system to an easier-to-solve form
   b. $A\mathbf{x} = \mathbf{b}$ becomes $U\mathbf{x} = \mathbf{d}$ and solution ensues (U is upper triangular)
   c. Method: row reduction using elementary row operations (EROs); Gaussian elimination or Gauss-Jordan elimination
      i. Multiply a row (j) by a value (c)
      ii. Add a multiple (c) of one row (j) to another (k)
      iii. Interchange rows j and k

<u>Example Problems in Solving Systems of Linear Equations</u>

Prob. 1:

$$3x_1 - x_2 + x_3 = -1$$
$$9x_1 - 2x_2 + x_3 = -9$$
$$3x_1 + x_2 - 2x_3 = -9$$

Prob. 2:

$$3x - y - 2z = 0$$
$$-6x + 2y + 6z = 4$$
$$2x + y + 6z = 13$$

Prob. 3:

$$x + 2y - 5z = 2$$
$$2x - 3y + 4z = 4$$
$$4x + y - 6z = 8$$

2. Going further..

   a. Upper triangular (matrix entries below main diagonal are zero)
   b. Row-echelon form (upper triangular with 1s on main diagonal)
   c. Reduced row-echelon form (rref in Matlab; EROs until identity matrix is obtained or can't go any further)

3. Singular systems: rank and consistency

   a. Consistent: has at least one solution or infinitely many solutions (other numbers of solutions – e.g., 3 – are not possible). Example in 2-D: Solution space is a point or line
   b. Inconsistent: No solutions: Example in 2-D: Parallel lines – no intersecting point
   c. Rank: Max. no. of independent row vectors in a matrix – examples coming soon

Adapted from lecture outline by Prof. James Maneval, Fall 2019