**ENGR 695 Advanced Topics in Engineering Mathematics Fall 2023**

**Lecture Outline for Monday, Oct. 30**

1. Not always possible to find a solution with SOV method; some PDE solutions are not separable. Which of the following PDEs can be solved via SOV, and which cannot?

   a. $x\dfrac{\partial u}{\partial x} = y\dfrac{\partial u}{\partial y}$

   b. $a^2\dfrac{\partial^2 u}{\partial x^2} - g = \dfrac{\partial^2 u}{\partial t^2}$, where $g$ is a constant

   c. $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y} + u$

   d. $a^2\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) = \dfrac{\partial u}{\partial t}$

2. Wave equation example

$$v^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 0 \le x \le L, \quad t \ge 0$$

   a. Vibrations of string: $v = \sqrt{\dfrac{T}{\rho}}$, where $T$ = tension in string, $\rho$ = mass per unit length

   b. Typical BCs and ICs (two ICs because time problem is second order)

$$u(0,t) = 0, \quad u(L,t) = 0, \quad u(x,0) = f(x), \quad \text{and} \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$$

   c. Apply SOV method to obtain

$$u(x,t) = \sum_{n=1}^{\infty}\left[A_n \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi v t}{L}\right) + B_n \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi v t}{L}\right)\right]$$

   with $A_n = \dfrac{2}{L}\int_0^L f(x)\sin\left(\dfrac{n\pi x}{L}\right)dx$ and $B_n = \dfrac{2}{n\pi v}\int_0^L g(x)\sin\left(\dfrac{n\pi x}{L}\right)dx$

   d. As with heat equation, BCs typically determine spatial eigenfunctions and ICs typically determine coefficients (use orthogonality of inner products)