

**What undergraduates can do:  
beginning, middle, and advanced research  
projects and experiences for undergraduates**

**Mathematics Teaching Day  
Banach Algebras 2007  
Université Laval, Québec City  
3 July 2007**

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# Why do research with undergrads?

1. Enhances student learning through mentoring relationships with faculty.
2. Creates relationships among students working on projects.
3. Increases retention in the science, technology, engineering and mathematics (STEM) pipeline.
4. Increases enrollment in graduate programs and provides effective career preparation.
5. Develops critical thinking, creativity, problem solving and intellectual independence.
6. Develops an understanding of research methodology, and introduces students to literature (journals, MathSciNet, etc.).
7. Promotes an innovation-oriented culture.

8. Increases breadth of projects in which faculty are involved.
9. Helps case for tenure, promotion.
10. Faculty member gains visibility in MAA circle.
11. Creates relationship between faculty and students, often lasts through and after graduate school.
12. All of the advantages students gain can have immediate payback in your classes.

# The Set Up

1. Finding the student;
2. Finding the problem;
3. Solving(?) the problem;
4. Talking about the work.
5. Writing up the work;

## Finding the student

1. Use the right student for the right problem.
2. Computer work/theoretical work/computational work.
3. Make your projects attractive to students...use pages.
4. Don't take someone if there's no one there.
5. If a student asks you, you don't have to say yes.

## Finding the problem

1. Choose problems you care about.
2. What would be my first step? No answer?  
No problem.
3. Good first steps are:
  - (a) Running some examples with *Mathematica*, *Matlab*, or *Java*;
  - (b) Computing things for low-degree polynomials or small matrices, numerical experiments, etc.

4. Which of these are good problems?

- (a) Given a rational function defined and continuous on the unit circle, find a function harmonic in the disk that equals the original on the boundary.
- (b) Let  $p$  be a polynomial with zeroes in the closed unit disk. Is it true that for each zero  $z_j$  of  $p$ , there is a zero  $a_j$  of the  $p'$  with  $|z_j - a_j| \leq 1$ ?

- (c) Let  $a$  and  $b$  satisfy  $|a| \leq 1, |b| \leq 1$  and

$$B(z) = z \left( \frac{z - a}{1 - \bar{a}z} \right) \left( \frac{z - b}{1 - \bar{b}z} \right)$$

be a Blaschke product. Then the line joining two points on  $|z| = 1$  identified by  $B$  is tangent to  $|z - a| + |z - b| = |1 - \bar{a}b|$ .

- (d) Show that the support set of an element of the maximal ideal space of  $H^\infty$  is a maximal antisymmetric set for  $H^\infty + C$ .
- (e) Given  $n$  distinct points  $z_1, \dots, z_n$  with  $|z_j| = 1$ , and  $w_1, \dots, w_n$  with  $|w_j| = 1$  construct a Blaschke product  $B$  such that  $B(z_j) = w_j$ .

5. Get them doing something right away.

<http://www.math.wustl.edu/~hjelle/>  
(Website. Geir Arne Hjelle,  
Washington University St. Louis)

6. Some other things students have done:

- (a) Making computer program to demonstrate Sendov conjecture;
- (b) Editing a textbook;
- (c) Writing a survey article of some overlooked but interesting problem.

7. Things do not always work out!

## Working on the problem

1. Start early.
2. Have students from previous projects advise new students.
3. Have your student read while working on the project.
4. If you have helpful colleagues around, try to have your student discuss with them and/or with each other.
5. Teach the students to use latex (handout).
6. Meet more at the beginning, less later.
7. Meet with your student frequently. Encourage your student.

# Talking about your work

1. The purpose of the talk.  
To impress? To overwhelm? To convey information?
2. How much technology should you use?  
Blackboard, Power point, Transparencies?
3. Know your audience.
4. Transitions and sign posts.
5. Decide what the central point is. Motivate it. Discuss the history. Put the theorems in context.
6. How to start the talk: Begin with
  - (a) an acknowledgement?
  - (b) outline?
  - (c) a joke? (students like this one)
  - (d) a question?
  - (e) an example?
  - (f) a description of how you (the student) got the problem?

7. Don't shoot too high. Explain all terms and keep definitions to a minimum.
8. (John E. McCarthy) "Prove only tautologies." (Do not give a proof that exceeds 5 lines.)
9. Make sure you keep the audience with you. How long?
10. Figure out how to end the talk.
11. Practice!! your talk! Listen to their talk before they give it. Criticize them.
12. Explain the role of the host.
13. Look at the room you will speak in.

## The write-up

1. Students need to know their audience!
2. Give your students experience in the classroom that will prepare them for research projects.
3. Have your students write other people's proofs up better than the original.
4. Students need to document their work, along the way.
5. Read the student's work carefully.
6. Make it clear to the student that, ultimately, responsibility for the paper rests on the author(s).

## The work's written, now what?

### 1. Conferences for students (present posters/talks):

- (a) Joint Mathematics Meetings (AMS/MAA);
- (b) Canadian Undergrad. Math. Conf.;
- (c) Nebraska Conf. for Undergrad. Women;
- (d) MathFest and MAA section meetings.

### 2. Publications

#### (a) MAA journals:

- i. College Mathematics Journal,
- ii. Mathematics Magazine,
- iii. American Mathematical Monthly,
- iv. Horizons.

#### (b) Journals targeting undergraduate authors

- i. Rose-Hulman Undergrad. Math. J.
- ii. Mathematical Spectrum
- iii. Studies in Mathematical Science

# Resources

## 1. Summer

(a) At your own school, with you:

- i. Internal funds,
- ii. your NSF grant (REU supplement),
- iii. REU Site,
- iv. CUR Undergrad. Summer Research Program

(b) Not at your school:

- i. REU Site,
- ii. NSA's Director's Summer Program,
- iii. Oak Ridge National Laboratory.

## 2. Academic year

Independent studies

(a) Students earn credit;

(b) You might too!

(c) Summer work (basis for honors theses during academic year).

# What are these REU programs?

1. REU Sites:  $\sim$  20 math REU Sites each summer.

(a) Facts:

- i. Not equal! Most “prestigious” (Joe Gallian (Duluth), SMALL (Williams)) are extremely competitive.
- ii. Some end with published research papers.
- iii. Students are encouraged to give talks at MAA or AMS conferences, or poster at Joint Meetings.
- iv. About 2 months, (students receive  $\sim$  \$3000).
- v. Faculty salary can be part of award.
- vi. Students must be US citizens or permanent residents.
- vii. Some students from your own school; must have ‘significant fraction’ from other schools.

- (b) Making your program/proposal a good one.
- i. House students together.
  - ii. Weekly seminars; students give short presentations and get feedback.
  - iii. Social program is required; make it attractive.
  - iv. Weekly social activities (ice cream social) for research students in all disciplines on campus.
  - v. Students make valuable contacts.
  - vi. Invite visiting speakers.
  - vii. Schools run poster sessions (Oct., May). Invite local newspaper.
  - viii. Try attract women and minorities. Discussing creative ways to do this (for example, recruitment trip to Spelman) can earn praise from NSF.

## 2. REU Supplements

- (a) Proposals may be included in new or renewal individual NSF grants or in any NSF-funded project, or through cooperative agreement with NSF.
- (b) Students must be US citizens or permanent residents.
- (c) Students from own school, so can hand-pick; usually for 1-3 students.

## Analysis at REU Sites

1. Properties of Toeplitz, Hankel, and Composition Operators (faculty: Jonathan Shapiro, Cal Poly);
2. Matrix Analysis and its Applications (faculty: Charles Johnson, College of William and Mary);
3. Matrix Analysis and Wavelet Theory (faculty: Dave Larson, Texas A & M);
4. Analysis on Fractals (faculty: Robert Strichartz, Cornell University);
5. Spontaneous Geometry (faculty: Ken Stephenson, University of Tennessee).

## Analysis at JMM (2007): Undergraduate Poster Session

1. Subnormality of Weighted Shifts (advisor: George Exner, Bucknell University);
2.  $L^p$  Boundedness of Oscillatory Integral Operators (advisor: Leslie Cheng, Bryn Mawr College);
3. Schur Multiplicative Maps on Matrices (advisor: Chi-Kwong Li, College of William and Mary);
4. Compactness and Measures of Noncompactness in Metric Trees (advisor: Asuman Aksoy, Claremont McKenna College);
5. Two Results Concerning Luzin's Theorem (advisor: Paul Humke, St. Olaf College);
6. Properties of Segments in the Hausdorff Metric Geometry (advisor: Steven Schlicker, Grand Valley State University).

In total, there were 169 posters.

## New Ideas?

The setting: A student is interested in doing research in your area, and you work at an undergraduate institution. What are some other ways to get students working with research program in analysis?

1. Send student on study abroad, to work with a connected research sponsor. Could we have a study abroad student work with both of us?
2. Create general partnerships with mathematicians at “research universities.”  
NSF program – the Research Opportunity Award – enables faculty at undergraduate institutions to visit research universities (summer, month, sabbatical). Take a student! (Or PEW Midstates Science Consortium)
3. Work more closely with disciplines that do this sort of thing well: engineers, physicists...

## The Disadvantages.

1. You don't always get paid.
2. It doesn't always work! Can be a waste of time and money (for you).
3. You won't always like your students. (See above...)
4. Trust: Is the work right?
5. Students do not like to do the documentation. They came to do research, not to write!
6. Changes your research.
7. You will need to criticize these students at some point. They aren't used to that.
8. Students *expect*: to do research, to have a paper, to work with you.
9. Students are young. You'll need to supervise them (some more/some less).

## References and Resources

1. National Science Foundation, REU  
[www.nsf.gov/crssprgm/reu/](http://www.nsf.gov/crssprgm/reu/)
2. Mathematical Association of America  
[www.maa.org/students/undergrad/](http://www.maa.org/students/undergrad/)
3. Math Archives, (somewhat outdated)  
<http://archives.math.utk.edu/undergraduates.html>
4. David Farmer, The AIM REU: individual projects with a common theme,  
[www.aimath.org/mathcommunity/farmerREU.pdf](http://www.aimath.org/mathcommunity/farmerREU.pdf)
5. David Farmer and Sally Koutsoliotas, Preparing Students to Give Talks,  
[www.aimath.org/mathcommunity/studenttalks.pdf](http://www.aimath.org/mathcommunity/studenttalks.pdf)
6. John E. McCarthy, *How to give a good Colloquium*, Canadian Mathematical Society Notes, Vol. 31 No. 5 [1999] 3-4  
[www.math.wustl.edu/~mccarthy/papers.html](http://www.math.wustl.edu/~mccarthy/papers.html)
7. Nebraska Conference for Undergraduate Women  
[www.math.unl.edu/~ncuwm](http://www.math.unl.edu/~ncuwm)
8. Canadian Undergraduate Mathematics Conference  
[cumc.math.ca/2007/en/Home/Home.html](http://cumc.math.ca/2007/en/Home/Home.html)

9. Oak Ridge National Laboratory  
[www.csm.ornl.gov/Internships/](http://www.csm.ornl.gov/Internships/)
10. Rose Hulman Institute of Technology Undergraduate Mathematics Journal,  
[www.rose-hulman.edu/mathjournal/](http://www.rose-hulman.edu/mathjournal/)
11. CUR (Council on Undergraduate Research),  
[www.cur.org](http://www.cur.org)

12. Senior Seminar description, SUNY Fredonia

[www.fredonia.edu/departments/math/MasterSyl/MATH/math405\\_master.html](http://www.fredonia.edu/departments/math/MasterSyl/MATH/math405_master.html)

How to evaluate talks (from the Fredonia web page):

- (a) The speaker exhibited the ability to organize data and analyze information, using appropriate mathematical tools.
- (b) Conjectures were formulated precisely. Counterexamples were used to disprove false conjectures; theorems were proved clearly and correctly.
- (c) The speaker showed the ability to learn mathematics independently, including the ability to refresh and deepen knowledge learned in previous courses.
- (d) The mathematics was communicated effectively, using proper notation and terminology.
- (e) Technology was used appropriately and with confidence to aid understanding.
- (f) The problem or purpose of the article was stated clearly.
- (g) The presentation was well prepared.
- (h) The presentation was well organized.
- (i) The background material and/or examples provided were relevant to the problem/article.

- (j) The mathematics used in the presentation was correct.
- (k) The logical development was clear.
- (l) The speaker showed an understanding of the definitions and terms used in the presentation.
- (m) The speakers use of voice enhanced the presentation and helped maintain audience interest.
- (n) The speaker maintained consistent eye contact with the audience.
- (o) The speaker involved the audience to an appropriate degree.
- (p) The speakers movements, gestures, appearance, and use of the blackboard and/or overhead projector contributed to the effectiveness of the presentation.
- (q) The presenter chose a problem/article and scheduled the presentation in a timely manner.
- (r) The presenters consultation with faculty was appropriate.