**H-4.1 **Elementary Discrete-Domain Functions (Sequences):**

Discrete-domain functions are defined for $n \in \mathbb{Z}$.

**H-4.1.1 Sequence Notation:**

We use the following notation to indicate the elements of a sequence $x[n]$ between index $n_L$ and index $n_H$:

$$x[n] = \{ x[n_L], x[n_L + 1], \ldots, x[n_H - 1], x[n_H] \}.$$

The elements outside of the given range are assumed to be zero (unless stated otherwise). The element that is associated with index $n = 0$ is indicated with an arrow:

$$x[n] = \{ \ldots, x[-2], x[-1], x[0], x[1], x[2], \ldots \} \uparrow.$$

If the arrow is omitted then the first given element in the sequence is assumed to be the element at index zero $x[n] = \{ x[0], x[1], x[2], \ldots \}$.

**H-4.1.2 Step Sequence:**

$$\mu[n] = \begin{cases} 
0 & \text{for } n < 0 \\
1 & \text{for } n \geq 0 
\end{cases}$$

**H-4.1.3 Kronecker Delta Sequence:**

$$\delta[n] = \begin{cases} 
1 & \text{for } n = 0 \\
0 & \text{for } n \neq 0 
\end{cases}$$

**H-4.2 Classification of Discrete-Domain Signals:**

We consider discrete-domain signals $x[n]$ that are defined for $n \in \mathbb{Z}$. The range of discrete-domain signals may be real ($x[n] \in \mathbb{R}$) or complex ($x[n] \in \mathbb{C}$).

**H-4.2.1 Periodic Signals:**

A discrete-domain signal $x[n]$ is periodic with period $N$ if there is a $N \in \mathbb{Z}$ such that $x[n] = x[n - N]$ for all $n \in \mathbb{Z}$.

**H-4.2.2 Symmetric Signals:**

A discrete-domain signal $x[n]$ is of even symmetry if $x[n] = x[-n]$. It is of odd symmetry if $x[n] = -x[-n]$. A (complex-valued) signal is of even Hermitian symmetry if $x[n] = x^*[n]$. It is of odd Hermitian symmetry if $x[n] = -x^*[n]$. 
H-4.2.3 Symmetry Decompositions:
A discrete-domain signal $x[n]$ can be decomposed into its
- even part $\frac{1}{2}(x[n] + x[-n])$
- odd part $\frac{1}{2}(x[n] - x[-n])$
- conjugate symmetric part $\frac{1}{2}(x[n] + x^*[−n])$ (even Hermitian symmetry)
- conjugate antisymmetric part $\frac{1}{2}(x[n] - x^*[−n])$ (odd Hermitian symmetry).

H-4.2.4 Bounded Signals:
A discrete-domain signal $x[n]$ is bounded if $|x[n]| \leq B_x < \infty$ for some finite $B_x \in \mathbb{R}^+$. (In writing $B_x$ we imply the smallest number such that $|x[n]| \leq B_x$.)

H-4.2.5 Energy Signals:
A discrete-domain signal $x[n]$ is an energy signal or square-summable signal if its energy $E_x$ is finite.
$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$$

H-4.2.6 Power Signals:
A discrete-domain signal $x[n]$ is a power signal if its power $P_x$ is finite.
$$P_x = \lim_{K \to \infty} \frac{1}{2K+1} \sum_{n=-K}^{K} |x[n]|^2 < \infty$$
A periodic signal $x[n]$ with period $N$ is a power signal with $P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$.

H-4.2.7 Absolutely Summable Signals:
A discrete-domain signal $x[n]$ is absolutely summable if
$$S_x = \sum_{n=-\infty}^{\infty} |x[n]| < \infty.$$ 

H-4.2.8 Finite Length Signals:
A discrete-domain signal $x[n]$ is of finite length if there exists a $n_1$ and a $n_2$ with $n_1 \leq n_2$ such that $x[n] = 0$ for all $n < n_1$ and $n > n_2$. Let $\tilde{n}_1$ denote the largest possible $n_1$ such that $x[n] = 0$ for all $n < \tilde{n}_1$ and let $\tilde{n}_2$ denote the smallest possible $n_2$ such that $x[n] = 0$ for all $n > \tilde{n}_2$ then the length of $x[n]$ is defined by:
$$L_x = \tilde{n}_2 - \tilde{n}_1 + 1.$$ 
Note that the length of a signal $x[n]$ that is identically equal to zero for all $n \in \mathbb{Z}$ is not defined!
H-4.2.9 Causal and Anti-Causal Signals:
A discrete-domain signal \( x[n] \) is causal if \( x[n] = 0 \) for all \( n < 0 \). It is anticausal if \( x[n] = 0 \) for all \( n > 0 \).

H-4.3 Elementary Discrete-Domain Signal Operations:
H-4.3.1 Convolution:
The discrete-domain convolution of two signals \( x[n] \) and \( h[n] \) is defined by
\[
y[n] = h[n] \ast x[n] = \sum_{k=-\infty}^{\infty} h[k] x[n - k].
\]
Convolution generally involves folding, shifting, multiplication, and summation.

H-4.3.2 Properties of Convolution:
The discrete-domain convolution operator \( \ast \) has the following properties:

a) Commutativity: \( x[n] \ast h[n] = h[n] \ast x[n] \)
b) Distributivity: \( x[n] \ast (h_1[n] + h_2[n]) = x[n] \ast h_1[n] + x[n] \ast h_2[n] \)
c) Associativity: \( (x_1[n] \ast x_2[n]) \ast x_3[n] = x_1[n] \ast (x_2[n] \ast x_3[n]) \)
d) Shift Property: \( h[n] \ast x[n] = y[n] \Rightarrow h[n-k] \ast x[n] = y[n-k] \)
e) Convolution Length: \( y[n] = h[n] \ast x[n] \Rightarrow L_y \leq L_h + L_x - 1 \)

H-4.3.3 Elementary Convolution Identities:

a) \( x[n] \ast \delta[n] = x[n] \) (i.e. \( x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n - k] \))
b) \( \mu[n] \ast \mu[n] = (n+1) \mu[n] \)

H-4.3.4 Properties of the Kronecker Delta Sequence:

a) Sum: \( \sum_{n=-\infty}^{\infty} \delta[n] = 1 \)
b) Exchange: \( x[n] \delta[n-k] = x[k] \delta[n-k] \)
c) Scaling: \( \delta[Kn] = \delta[n] \) for \( K \in \mathbb{Z} \)
d) Convolution: \( x[n] \ast \delta[n-k] = x[n-k] \)
e) Symmetry: \( \delta[n] = \delta[-n] \)

H-4.3.5 Deterministic Correlation:
The (deterministic) correlation of two energy signals \( x[n] \) and \( y[n] \) is defined by
\[
r_{xy}[k] = \sum_{n=-\infty}^{\infty} x[n+k] y^*[n] = x[k] \ast y^*[-k].
\]
For two power signals \( x[n] \) and \( y[n] \) we define respectively
\[
\tilde{r}_{xy}[k] = \lim_{K \to \infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x[n+k] y^*[n].
\]
For two signals $x[n]$ and $y[n]$ that are both periodic with period $N$ we obtain

$$\hat{r}_{xy}[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n+k] y^*[n].$$

**H-4.4 Classification of Discrete-Domain Systems:**

We consider discrete-domain systems $\mathcal{T}$ with input $x[n]$ and output $y[n]$.

$$y[n] = \mathcal{T}\{x[n]\}$$

**H-4.4.1 Linear Systems:**

A discrete-domain system $\mathcal{T}$ is linear if for any two arbitrary input signals $x_1[n]$, $x_2[n]$ and for any two constants $\alpha_1, \alpha_2 \in \mathbb{R}$ (or $\mathbb{C}$) we have

$$\mathcal{T}\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\} = \alpha_1 \mathcal{T}\{x_1[n]\} + \alpha_2 \mathcal{T}\{x_2[n]\}.$$

**H-4.4.2 Time-Invariant Systems:**

A discrete-domain system $\mathcal{T}$ is time-invariant if $y[n] = \mathcal{T}\{x[n]\}$ implies that $y[n-k] = \mathcal{T}\{x[n-k]\}$ for any arbitrary input signal $x[n]$ any arbitrary delay $k \in \mathbb{R}$.

**H-4.4.3 Causal Systems:**

A discrete-domain system $\mathcal{T}$ is causal if the output $y[n]$ at time $n$ only depends on current and past input values $x[k]$ for $k \leq n$ and/or only depends on past output values $y[k]$ for $k < n$.

**H-4.4.4 BIBO Stable Systems:**

A discrete-domain system $\mathcal{T}$ is bounded-input bounded-output (BIBO) stable if any bounded input $|x[n]| \leq B_x < \infty$ leads to a bounded output $|y[n]| \leq B_y < \infty$.

**H-4.4.5 Passive and Lossless Systems:**

A system with arbitrary square summable input $x[n]$ and output $y[n]$ is called passive if $\mathcal{E}_y \leq \mathcal{E}_x$. Systems for which $\mathcal{E}_y = \mathcal{E}_x$ for any square summable input $x[n]$ are called lossless.

**H-4.4.6 Up-Sampling and Down-Sampling Systems:**

A discrete-domain system that inserts $L-1$ ($L \in \mathbb{N}$) zeros between every element of an input sequence $x[n]$ is called an up-sampling system of order $L$:

$$y[n] = \begin{cases} x[n/L] & \text{for } n = Lk \text{ with } k \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

A discrete-domain system is called a down-sampling system of order $L$ if it discards all elements of input $x[n]$ that are not indexed by a multiple of $L$:

$$y[n] = x[n \cdot L]$$
H-4.5  **Discrete Linear Time-Invariant (DLTI) Systems:**

H-4.5.1  **Impulse Response:**

Let $T$ denote a DLTI system. If we let the *impulse response* $h[n]$ of $T$ be defined as $h[n] = T\{ \delta[n] \}$ then the response of $T$ to an arbitrary input $x[n]$ is given by

$$y[n] = x[n] \ast h[n].$$

H-4.5.2  **Causal DLTI Systems:**

A DLTI system $T$ is *causal* if and only if its impulse response $h[n]$ is a causal signal:

$$h[n] = 0 \text{ for } n < 0.$$ 

H-4.5.3  **BIBO Stable DLTI Systems:**

A DLTI system $T$ is *BIBO stable* if and only if its impulse response $h[n]$ is absolutely summable, i.e. if $\mathcal{S}_h < \infty$.

H-4.5.4  **FIR and IIR Systems:**

A DLTI system is called a *finite impulse response system* (FIR system) if the length of the impulse response $h[n]$ is finite, i.e. if $\mathcal{L}_h < \infty$. A DLTI system is called an *infinite impulse response system* (IIR system) if $\mathcal{L}_h = \infty$.

H-4.5.5  **Eigenfunctions of DLTI Systems:**

Input functions of the form $x[n] = z_0^n$ are *eigenfunctions* of DLTI systems.

$$y[n] = T\{ z_0^n \} = h[n] \ast z_0^n = z_0^n \cdot \sum_{k=-\infty}^{\infty} h[k] z_0^{-k} = z_0^n \cdot H(z_0)$$

When passed through a DLTI system, these eigenfunctions remain unchanged up to a constant (possibly complex) gain $H(z_0)$.

H-4.6  **The Z-Transform:**

H-4.6.1  **Definition of the (Bilateral) Z-Transform:**

The *(bilateral)* $z$-transform $X(z)$ of signal $x[n]$ is defined by

$$X(z) = \mathcal{Z}\{ x[n] \} = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

with $\text{ROC}: 0 \leq r_1 < |z| < r_2 \leq +\infty$.

The $z$-transform always consists of both the complex function $X(z)$ and its associated *region of convergence* (ROC). The region of convergence is the set of all complex values $z$ for which the transform summation converges. The ROC is generally a ring in the complex plane, bounded by an inner radius $r_1$ and an outer radius $r_2$ ($r_1, r_2 \in \mathbb{R}^+$). The radius $r_1$ is determined by the rate of exponential increase/decrease of the causal part of $x[n]$. Similarly, $r_2$ is determined by the rate of exponential increase/decrease of the anti-causal part of $x[n]$. 

H-4.6.2 The Inverse Z-Transform:
The inverse z-transform is defined as
\[ x[n] = Z^{-1}\{X(z)\} = \frac{1}{2\pi j} \oint_{C} X(z) z^{n-1} \, dz \]
in which the integration contour \( C \) is given by
\[ C: r e^{j\omega} \mid \begin{align*}
\omega &= +\pi \\
\omega &= -\pi
\end{align*} \text{ for some fixed } r \in ]r_1, r_2 [. \]

H-4.6.3 Complex Contour Integration:
If a sufficiently smooth complex contour \( C \) can be described with a parameter description \( p(\varphi) \in \mathbb{C} \) for \( \varphi \in [a, b] \) then \( \int_{C} F(s) \, ds = \int_{a}^{b} F(p(\varphi)) p'(\varphi) \, d\varphi \). A complex contour integral can thus be reduced to a conventional Riemann integral.

H-4.6.4 Five Elementary Z-Transform Identities:

<table>
<thead>
<tr>
<th>( x[n] = Z^{-1}{X(z)} )</th>
<th>( X(z) = Z{x[n]} )</th>
<th>ROC:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta[n] )</td>
<td>1</td>
<td>( z \in \mathbb{C} )</td>
</tr>
<tr>
<td>( \alpha^n \mu[n] )</td>
<td>( \frac{z}{z-\alpha} )</td>
<td>(</td>
</tr>
<tr>
<td>( -\alpha^n \mu[-n-1] )</td>
<td>( \frac{z}{z-\alpha} )</td>
<td>(</td>
</tr>
<tr>
<td>( n \alpha^n \mu[n] )</td>
<td>( \frac{\alpha z}{(z-\alpha)^2} )</td>
<td>(</td>
</tr>
<tr>
<td>( -n \alpha^n \mu[-n-1] )</td>
<td>( \frac{\alpha z}{(z-\alpha)^2} )</td>
<td>(</td>
</tr>
</tbody>
</table>

H-4.6.5 The Z-Transform of Causal Signals:
Note that every valid z-transform expression \( X(z) \) has only one causal inverse transform \( x[n] \). We do not need to know the ROC explicitly to find the correct causal inverse of \( X(z) \).
### H-4.6.6 A Short Table of Z-Transforms of Causal Signals:

<table>
<thead>
<tr>
<th>Signal</th>
<th>Z-Transform</th>
<th>ROC:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x[n] = Z^{-1}{X(z)}$</td>
<td>$X(z) = Z{x[n]}$</td>
<td>$</td>
</tr>
<tr>
<td>$\mu[n]$</td>
<td>$\frac{z}{z-1}$</td>
<td>$</td>
</tr>
<tr>
<td>$\alpha^n \cos(\omega_0 n) \mu[n]$</td>
<td>$\frac{z^2 - \alpha z \cos \omega_0}{z^2 - 2\alpha z \cos \omega_0 + \alpha^2}$</td>
<td>$</td>
</tr>
<tr>
<td>$\alpha^n \sin(\omega_0 n) \mu[n]$</td>
<td>$\frac{\alpha z \sin \omega_0}{z^2 - 2\alpha z \cos \omega_0 + \alpha^2}$</td>
<td>$</td>
</tr>
</tbody>
</table>

### H-4.6.7 Properties of the Bilateral Z-Transform:

<table>
<thead>
<tr>
<th>Operation</th>
<th>$x[n] = Z^{-1}{X(z)}$</th>
<th>$X(z) = Z{x[n]}$ and ROC$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>$\alpha_1 x_1[n] + \alpha_2 x_2[n]$</td>
<td>$\alpha_1 X_1(z) + \alpha_2 X_2(z)$ ROC$_1 \cap$ ROC$_2$</td>
</tr>
<tr>
<td>Time Shift</td>
<td>$x[n-k]$</td>
<td>$X(z) z^{-k}$ and same ROC$^b$</td>
</tr>
<tr>
<td>Modulation</td>
<td>$\alpha^n x[n]$</td>
<td>$X(z/\alpha)$ ROC$^c$ is scaled by $</td>
</tr>
<tr>
<td>Differentiation in Z-Domain</td>
<td>$n x[n]$</td>
<td>$-z \frac{d}{dz} X(z)$ and same ROC</td>
</tr>
<tr>
<td>Conjugation</td>
<td>$x^*[n]$</td>
<td>$X^<em>(z^</em>)$ and same ROC</td>
</tr>
<tr>
<td>Convolution</td>
<td>$x[n] \ast h[n]$</td>
<td>$X(z) \cdot H(z)$ ROC$_1 \cap$ ROC$_2$</td>
</tr>
</tbody>
</table>

$^a$The actual ROC of the result of an operation may be larger than the one provided in the table. Check the common literature on z-transforms for the details.

$^b$Same ROC possibly except $z = 0$ if $k > 0$.

$^c$If the original ROC of $X(z)$ is given by $r_1 < |z| < r_2$ then the scaled ROC of $X(z/\alpha)$ is given by $|\alpha| r_1 < |z| < |\alpha| r_2$
H-4.7 **DLTI Systems and the Z-Transform:**

**H-4.7.1 Transfer Functions and BIBO Stable Systems:**

Let $H(z) = \mathcal{Z}\{h[n]\}$ denote the z-transform of the impulse response $h[n]$ of a DLTI system. $H(z)$ is called the *transfer function* of the DLTI system. A DLTI system is *BIBO stable* if the unit circle ($|z| = 1$) is contained in the ROC of its transfer function $H(z)$.

**H-4.7.2 Linear Constant Coefficient Difference Equations:**

Every linear constant coefficient difference equation with input $x[n]$ and output $y[n]$ establishes a causal linear time-invariant system.

$$y[n] = -a_1 y[n-1] - a_2 y[n-2] - \ldots$$

$$\ldots - a_N y[n-N] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots$$

$$\ldots + b_M x[n-M]$$

By transforming the difference equation into the z-domain we obtain the transfer function $H(z)$ of the associated DLTI system. The transfer function of a linear constant coefficient difference equation is rational in variable $z$:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N}}$$

Since $H(z)$ is the transfer function of a causal system we do not need to explicitly provide its ROC. Furthermore, we can write every rational transfer function of the form above in terms of its poles $p_i$ (for $i = 1 \ldots N$) and zeros $z_i$ (for $i = 1 \ldots M$).

$$H(z) = b_0 \cdot z^{(N-M)} \cdot \frac{(z-z_1)(z-z_2)\ldots(z-z_M)}{(z-p_1)(z-p_2)\ldots(z-p_N)}$$

The term $b_0$ is often referred to as the *gain* of the system. Note, however, that $b_0$ is usually *not* equal to the DC gain or the high-frequency gain of a system!

**H-4.7.3 Stability of Causal DLTI Systems with Rational Transfer Functions:**

A causal DLTI system with a rational transfer function $H(z)$ is stable if and only if the magnitude of all of its poles is strictly smaller than one ($|p_i| < 1$ for $i = 1 \ldots N$), i.e. if all poles are strictly inside of the unit circle.

**H-4.7.4 System I/O Description in the Z-Domain:**

Due to the convolution theorem of the z-transform we can find the output $y[n]$ of a DLTI system for a given input $x[n]$ conveniently in the Z-Domain:

$$Y(z) = \mathcal{Z}\{y[n]\} = H(z) \cdot X(z) = \mathcal{Z}\{h[n]\} \cdot \mathcal{Z}\{x[n]\}.$$ 

If $Y(z)$ is rational then we can find its inverse transform $y[n]$ via a partial fraction expansion in $z^{-1}$ and a table lookup.
H-4.8 **The Discrete-Time Fourier Transform (DTFT):**

H-4.8.1 **Definition of the Discrete-Time Fourier Transform:**

The *discrete-time Fourier transform* (DTFT) and its inverse are defined by

\[
X(\omega) = \text{DTFT}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}
\]

and

\[
x[n] = \text{DTFT}^{-1}\{X(\omega)\} = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega.
\]

The existence of the discrete-time Fourier transform is guaranteed for absolutely summable signals. For other signals meaningful definitions for the DTFT may be found, but the existence is not guaranteed in general.

H-4.8.2 **Some Elementary DTFT Identities:**

<table>
<thead>
<tr>
<th>(x[n] = \text{DTFT}^{-1}{X(\omega)})</th>
<th>(X(\omega) = \text{DTFT}{x[n]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x[n] = 1)</td>
<td>(X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k))</td>
</tr>
<tr>
<td>(x[n] = \delta[n - k])</td>
<td>(X(\omega) = e^{-j\omega k})</td>
</tr>
<tr>
<td>(x[n] = e^{j\omega n})</td>
<td>(X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k))</td>
</tr>
<tr>
<td>(x[n] = \mu[n])</td>
<td>(X(\omega) = \frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k))</td>
</tr>
<tr>
<td>(x[n] = \alpha^n \mu[n]\text{ with }</td>
<td>\alpha</td>
</tr>
<tr>
<td>(x[n] = \begin{cases} 1 &amp; \text{for }</td>
<td>n</td>
</tr>
<tr>
<td>(x[n] = \begin{cases} \omega_0/\pi \frac{\sin(\omega_0 n)}{\pi n} &amp; \text{for } n = 0 \ \frac{1}{\pi n} &amp; \text{for } n \neq 0 \end{cases})</td>
<td>(\hat{X}(\omega) = \begin{cases} 1 &amp; \text{for }</td>
</tr>
</tbody>
</table>

| \(X(\omega) = \sum_{k=-\infty}^{\infty} \hat{X}(\omega - 2\pi k)\) |

Note that we can directly derive the DTFT $X(\omega)$ of a signal $x[n]$ from its z-transform $X(z)$ if the ROC of $X(z)$ contains the unit circle.

$$X(\omega) = X(z) \mid _{z=e^{j\omega}} \text{ if } e^{j\omega} \in \text{ROC} \text{ for } \omega \in [-\pi, \pi]$$

There is an ambiguity in our notation for the z-transform $X(z)$ and the DTFT $X(\omega)$. The distinction is achieved with the name of the independent variable: ($z$) for the z-transform and ($\omega$) for the DTFT.

### H-4.8.3 Properties of the DTFT:

<table>
<thead>
<tr>
<th>Operation</th>
<th>$x[n] = \text{DTFT}^{-1}{X(\omega)}$</th>
<th>$X(\omega) = \text{DTFT}{x[n]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>$\alpha_1 x_1[n] + \alpha_2 x_2[n]$</td>
<td>$\alpha_1 X_1(\omega) + \alpha_2 X_2(\omega)$</td>
</tr>
<tr>
<td>Time Shift</td>
<td>$x[n - k]$</td>
<td>$X(\omega) e^{-j\omega k}$</td>
</tr>
<tr>
<td>Frequency Shift</td>
<td>$x[n] e^{j\omega_0 n}$</td>
<td>$X(\omega - \omega_0)$</td>
</tr>
<tr>
<td>Time Reversal</td>
<td>$x[-n]$</td>
<td>$X(-\omega)$</td>
</tr>
<tr>
<td>Conjugation</td>
<td>$x^*[n]$</td>
<td>$X^*(-\omega)$</td>
</tr>
<tr>
<td>Frequency</td>
<td>$nx[n]$</td>
<td>$j \frac{\partial}{\partial \omega} X(\omega)$</td>
</tr>
<tr>
<td>Differentiation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Convolution</td>
<td>$x[n] \circledast h[n]$</td>
<td>$X(\omega) \cdot H(\omega)$</td>
</tr>
<tr>
<td>Cross-Correlation</td>
<td>$x[n] \circledast y^*[-n]$</td>
<td>$X(\omega) \cdot Y^*(\omega)$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$x[n] \cdot y[n]$</td>
<td>$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(\lambda)Y(\omega - \lambda) d\lambda$</td>
</tr>
</tbody>
</table>